THE IMPACT OF STATISTICS AND MANAGEMENT SCIENCE TECHNIQUES UPON THE CONSTRUCTION AND UTILIZATION OF STANDARD MANUFACTURING COSTS

By

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This study analyzes the impact of statistical and management science techniques upon manufacturing cost standards -- their construction and utilization Particular emphasis is placed upon the areas of the setting of labor quantity standards, the separation of mixed overhead costs into their fixed and variable components, variance analysis, joint product cost allocation, and service department cost allocation Only the impact of quantitative procedures has been considered.

The techniques which are discussed include learning curves, regression analysis, control charts, modern decision theory, controlled cost, matrix algebra, and linear programming These procedures are reviewed briefly as to their method of application following which their impact is analyzed In some cases, where deemed pertinent, examples of the application of a particular technique or the interpretation of the results,

have been presented, e.g., learning curves used in construction of labor standards

In general the impact of these techniques appears to be varied. Learning curves may be employed to install a dynamic element in the establishment of labor time and cost standards. Control charts and modern decision theory have moved the viewing of a standard from that as a single fixed point estimate to a range. In addition, modern decision theory expands the parameters of variance analysis adding such elements as investigative cost and opportunity cost.

Techniques such as controlled cost or linear programming, both of which are suggested for use in the area of variance analysis and control, appear to have had more of an impact upon general thinking in the area rather than specifically having an impact upon practice or text presentation. The utilization of matrix algebra in the allocation of service department costs is reviewed and appears to have been utilized mainly as a computational tool at the present time. Regression analysis, which was suggested for use in three areas: separation of fixed and variable costs into their fixed and variable elements, the allocation of joint-product costs, and variance analysis, also appears to have had an initial impact as a computational device but, based upon interpretation of the results, a potential conceptual impact is likely. Statistical and management science techniques are bringing in an increased sophistication to the construction and utilization of standard costs.

I INTRODUCTION

The greatest impetus to the development of standard costing occurred in the early twentieth century mainly through the work of engineers rather than accountants. A number of histories, or historical references, have appeared which deal with the development of standard costing through 1935.[1] The early work in standard costing was carried out along two tracks: 1) by efficiency engineers who were mainly interested in the elimination of industrial waste through cost control, and 2) by accountants who were aiming at the discovery of "true costs."[2]

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[1]Some of these histories and historical references are: Ellis Mast Sewell, The Evolution of the Theories and Techniques of Standard Costs (Ph.D. Dissertation, University of Texas at Austin, 1944) which surveys the historical development through G. Charter Harrison; Vernon Hill Upchurch, The Contributions of G. Charter Harrison to Cost Accounting (Ph.D. Dissertation, University of Texas at Austin, 1954), especially Chapter II; S. P. Garner, Evolution of Cost Accounting to 1925 (Alabama: University of Alabama Press, 1954) which contains some scattered references to standard costing; Karl Weber, Amerikanische Standardkostenrechnung Ein Uberblick (Winterthur: P. G. Keller, 1960) which is a brief survey of the accounting literature in America from 1900 to about 1960; David Solomons, "The Historical Development of Costing," in Studies in Costing. Ed. David Solomons (London: Sweet & Maxwell, Limited, 1952); Kiyoshi Okamoto, "Evolution of Cost Accounting in the United States of America (II)," Hitotsubashi Journal of Commerce and Management (April, 1968), pp. 28-34.

[2]Okamoto, p. 28.

The difference in the two approaches was emphasized by Castenholz in 1922. He set up two types of standards: cost and production, which were different in both construction and use but which should approach each other in quantitative terms as closely as possible.[3] No attempt was made at this time, however, to utilize these standards in a cost accounting system.[4] The clearest and most modern, presentation of standard costing appeared in the writings of G. Charter Harrison, many of which "are still part of [the] current literature" on cost accounting.[5]

Standard costing is an important branch of cost accounting as was noted by the Institute of Chartered Accountants in England and Wales:

> In our view standard costing is a most important development in accounting techniques, which enables the accountant to provide management with vital information essential for the day-to-day control of a manufacturing organization. As such it merits the closest study, not only by accountants engaged in industry but also by practising accountants who are or may be required to advise their clients on the subject of cost accounting.[6]

Despite this view of the significance of standard costing, very few books have been written which are devoted solely to standard costing, its tech-

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[3]Ibid., p. 32. The cost standard was an expression of "assumed normal experience results," whereas the production standards were "based upon an operating ideal and [became] indices of operating efficiency."

[4]Ibid.

[5]Solomons, p. 50.

[6]Developments in Cost Accounting, Institute of Chartered Accountants in England and Wales. Report of the Cost Accounting Sub-Committee of the Taxation and Financial Relations Committee, 1955, as quoted by Weber, p. 340.

niques, development or application.[7] The topic, however, is [illegible] as a separate section in most textbooks on cost accounting.

Much was published in the literature regarding standard costs during the first three decades of the twentieth century but, by the end of the 1930's, enthusiasm for standard cost accounting began to wane in favor of actual cost job-cost systems. This move coincided with the beginning of the second world war which created an emphasis on the cost of jobs and contracts and pushed the standard cost literature into a temporary period of 'stagnation.'[8]

In the last several decades a growing interest in the areas of management science and statistics has developed. This is evidenced in college curricula as well as in practice. More and more students of business administration are being exposed to the basic concepts, at least, of statistics and management science in their undergraduate and/or graduate programs.[9] This increasing interest is also apparent in the various accounting periodicals, leading to a frequent complaint

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[7]See for example, J. Batty, Standard Costing (3rd ed.; London: Macdonald and Evans, Ltd., 1970), Stanley B. Henrici, Standard Costs for Manufacturing (3rd ed., New York: McGraw-Hill Book Company, 1960), Cecil Gillespie, Standard and Direct Costing (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962), Clinton W. Bennett, Standard Costs ... How They Serve Modern Management (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957). Two earlier books in this area are: G. Charter Harrison, Standard Costs (New York: The Ronald Press, Co., 1930) and Eric A. Camman, Basic Standard Costs (New York: The American Institute Publishing Company, 1932).

[8]Weber, p. 211.

[9]For example, Florida International University is requiring as part

that their articles no longer are concerned with accounting.[10]

Two cogent reasons may be given for the need for an inquiry into the effect of statistical and management science techniques on standard costing. First, some of the more recent textbooks on cost accounting include sections on various statistical and management science techniques,[11] and second, a number of suggested applications of statistical and management science models to various areas of standard cost accounting problems or procedures have appeared in the periodical literature of the last twenty years and especially in the last decade. The textbook references have carried general discussions concerning the mechanics of techniques rather than relating them to a specific aspect of cost accounting, e.g., standard costs. The emphasis has been on their use for the separation of mixed costs into their fixed and variable components, cost control, or cost allocation, all of which are integral parts of standard costing. The statistical and management science

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of the core courses required of all business majors at the present time are course each in statistics, operations management, and information systems.

[10]Evidence of this problem is a recent survey taken by the American Accounting Association regarding the types of articles its members would prefer to see in The Accounting Review; results are unavailable at present.

[11]See for example: Charles T. Horngren, Cost Accounting: A Managerial Emphasis (3rd ed.; Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1972); Gerald R. Crowningshield, Cost Accounting Principles and Managerial Application (2nd ed.; Boston: Houghton Mifflin Company, 1969); Nicholas Dopuch and Jacob G. Birnberg, Cost Accounting: Accounting Data for Management's Decisions (Chicago: Harcourt, Brace & World, Inc., 1969); Gordon Shillinglaw, Cost Accounting Analysis and Control (3rd ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1972).

models considered in the periodicals often are related to the results of a specific application of one of the various techniques mentioned in the textbooks to standard costing problems, but these discussions vary between generalized considerations of the applicability of a particular process, possibly using a hypothetical set of data, and specific discussion of the results obtained when a technique has been tested in an actual situation. Nowhere, however, does there appear to be any discussion which looks at all the procedures suggested for particular applications, their advantages and disadvantages.

Methodology

The basis for the information in this study will be a number of references contained in periodicals, books and several recent dissertations, all of which deal with areas of cost accounting, statistics and/or management science. Various statistical and management science techniques which are in current use or have been suggested for use in conjunction with standard costing will be discussed and evaluated as to their impact. A reverse situation, the application of standard costs and quantities as input coefficients for linear programming models will also be considered. Finally, possible trends in the development of standard costing will be explored.

It is difficult to develop criteria for differentiating between those techniques suggested for use and those which are in actual use. Some techniques have been discussed in the literature for a great number of

years (e.g., control charts) while others have been developed for use in a particular firm but apparently do not appear to be in general use (e.g., _ex post_ variance analysis). Other techniques are discussed in the literature which apparently have no basis in practice (e.g., controlled cost).

Definitions

Standard Costs

A number of definitions of "standard cost" are posed in the accounting literature. In general, standard costs may be compared to a benchmark,[12] or to a criterion to be used to measure and appraise manufacturing costs, marketing costs, and occasionally, clerical costs.[13] The standard emphasizes what costs, or quantities, should be in a particular situation.[14]

The concept of standard costs is closely related to, and dependent upon, the idea of standard quantities, times and methods. A definition of a standard given in 1934 is:

> A standard under the modern scientific movement is simply a carefully thought out method of performing a function or carefully drawn specification covering an implement or some article of store or of product. . . . The standard method of doing any-

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[12]Henrici, p. 5.

[13]S. Winston Korn and Thomas Boyd, _Accounting for Management Planning and Decision Making_ (New York: John Wiley & Sons, Inc., 1969), p. 502.

[14]Henrici, p. 8.

> thing is simply the best method that can be devised at the time the standard is drawn.[15]

The standard cost for a product or operation is determined by pricing the engineering specifications for labor, material and overhead at predetermined basic rates.[16]

A more expanded and current definition of a standard cost is the following:

> [A standard cost is] a forecast or predetermination of what costs should be under projected conditions, serving as a basis of cost control, and as a measure of productive efficiency when ultimately compared with actual costs. It furnishes a medium by which the effectiveness of current results can be measured and the responsibility for deviations can be placed. A standard cost system lays stress upon important exceptions and permits concentration upon inefficiencies and other conditions that call for remedy.[17]

Various types of standard cost systems have been suggested and operated during the fifty years since the first standard cost system was put into use by G. Charter Harrison.[18] Regardless of the type of standard cost used, standard costing should not be viewed as a separate system of cost accounting but as one which may be integrated into either

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[15] Morris L. Cooke, quoted in Cost and Production Handbook, Ed. L. P. Alford (New York: The Ronald Press Company, 1934), as quoted in Upchurch, p. 19.

[16] Camman, p. 34.

[17] Bennett, p. 1.

[18] Wilmer Wright, Direct Standard Costs for Decision Making and Control (New York: McGraw Hill Book Company, Inc., 1962), p. 4. The systems differed generally in the type of standard used (bogey, ideal, expected actual, etc.) and how it was integrated into the system

the job order or the process cost system.[19] Standard costing "merely establishes maximum levels of production costs and efficiency."[20]

Standard costs may be employed to achieve a variety of purposes. One writer states that they may be used to achieve:

1. Efficient planning and budgeting
2. Control over costs with a view to conforming their amounts to those envisaged in the profit control plan.
3. Motivation of personnel in a variety of ways, to reduce costs, to increase output, and more fully to utilize facilities
4. Preparation of financial statements
5. Convenience in accounting for inventories and costs
6. Pricing of products, present or prospective.
7. Motivation of the appropriate level of management to provide the most efficient equipment
8. Making of appropriate decisions in situations involving alternative actions
9. Establishment of uniform prices for an industry.[21]

Statistical and management science techniques to be discussed in the following chapters in general are aimed at improving the standards utilized to secure the foregoing, especially the second, third and fifth purposes.

Statistics and Probability

The term "statistics" is used in the title of this study, but two terms actually need to be considered, "statistics" and "probability" since probability theory is essential to statistical inference which plays a prominent role in several of the statistical models to be discussed.

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[19]Korn and Boyd, p. 502. [20]Ibid.

[21]Lawrence J. Benninger, "Utilization of Multi-Standards in the Expansion of an Organization's Information System," Cost and Manage-

Statistical inference and probability, although related, function in counter-directions. Probability theory may be compared to the deductive method of reasoning in that the model is used to deduce the specific properties of the physical process while statistical inference more closely resembles inductive reasoning since the properties of the model are inferred from the data.[22] Statistics, then, is used to help the decision maker reach wise decisions in the face of uncertainty while probability theory is more concerned with studying "the likelihood of an event's happening."[23]

One branch of statistics which will be of prime importance in the area of cost control is statistical decision theory which "incorporates the decision maker's reaction to the occurrence of all possible events for each possible act."[24] A decision rule is then applied to the evaluation of the evidence in order to choose the best act.[25] A number of decision rules exist, but Bayes' decision rule is the one which is widely

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ment (January February, 1971), p. 24.

[22]Thomas H. Williams and Charles H. Griffin, The Mathematical Dimension of Accountancy (Chicago: South-Western Publishing Co., 1964), p. 135.

[23]David H. Li, Cost Accounting for Management Applications (Columbus, Ohio: Charles E. Merrill Books, Inc., 1966), p. 608.

[24]Harold Bierman, Jr., "Probability, Statistical Decision Theory and Accounting," The Accounting Review, XXXVII (July, 1962), p. 401.

[25]Ibid.

supported as being applicable to a broad variety of problems.[26]

Bayes' theorem, which forms the basis for Bayes' decision rule, requires the use of two types of probabilities, prior and posterior. The prior probabilities are probabilities which are "assigned to the values of the basic random variable before some particular sample is taken"; posterior probabilities are the prior probabilities which have been revised to take into account the additional information which has been provided by the sample.[27] If a subsequent sample is taken, these posterior probabilities act as new prior probabilities.

Generally there are two pieces of information developed when a problem is formulated in a Bayesian inference model. The first is a payoff table which shows the acts, events and utilities for each combination of act and event; the second is the probability distribution for the events. These are then used to calculate the expected utility for each act, and the act with the maximum utility is chosen.[28] Bayesian analysis is most useful to the accountant in the provision of a quantitative methodology by which prior intuitive knowledge may be included in an analysis, e.g.,

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[26] Ibid. Some of the other possible decision rules mentioned by Bierman are: Minimax, Maximax, Maximum Likelihood and Equally Likely.

[27] Robert Schlaifer, Probability and Statistics for Business Decisions (New York: McGraw-Hill Book Company, Inc., 1959), p. 337.

[28] Harry V. Roberts, "Statistical Inference and Decision" (Syllabus, University of Chicago, Graduate School of Business, 1962), p. 10-1.

an analysis of cost variances from budget.[29]

> the probabilities of a Bayesian prediction (1) are attached directly to the possible outcomes of a future sample and (2) are not conditional on unknown parameters, although they are conditional on prior distributions.[30]

Morris Hamburg distinguished between classical and Bayesian statistics as follows:

> . . . in classical statistics, probability statements generally concern conditional probabilities of sample outcomes given specified population parameters. The Bayesian point of view would be that these are not the conditional probabilities we are usually interested in. Rather we would like to have the very thing not permitted by classical methods -- conditional probability statements concerning population values, given sample information.[31]

The testing of hypotheses also differs under Bayesian decision theory. Under traditional testing methods, prior information is not combined with experimental evidence, and the decisions made between alternative acts is based solely upon significance levels. Under Bayesian decision theory, prior and sample data are combined and the "economic costs" of choosing one alternative over another are included in the decision

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[29] J. G. Birnberg, "Bayesian Statistics: A Review," The Journal of Accounting Research, II (Spring, 1964), p. 111.

[30] Harry V. Roberts, "Probabilistic Prediction" (unpublished paper, University of Chicago, April, 1964), p. 3. The formula for Bayes' theorem may be expressed in words as follows:

$$\text{Prior density of parameters, given sample} = \frac{(\text{Prior density of parameters})(\text{Likelihood function of sample})}{\text{Prior density of sample}}$$

[31] Morris Hamburg, "Bayesian Decision Theory and Statistical Quality Control," Industrial Quality Control (December, 1962), p. 11.

process.[32]

Management Science

There have been two views as to what management science is, or where it stands in relation to the more familiar term "operations research." The first of these views was expressed by Dantzig who said: "Operations Research or Management Science, two names for the same theory, refers to the science of decision and its applications."[33] This view is repeated by Simon: "No meaningful line can be drawn to demarcate operations research from scientific management or scientific management from management science."[34]

The other, opposing, view of management science was expressed by Symonds who differentiated between operations research and management science as follows:

> Application of the scientific method to specific problem solving in the area of management is called operations research. Operations research uses scientific principles and methods in solving specific problems. Operations research study does not usually produce general laws or fundamental truths. Although operations research and management science are now closely related, they are quite different but complementary in their purposes. Operations research represents the problem-solving objective; management science the development of general scien-

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[32] *Ibid.*, p. 14.

[33] George B. Dantzig, "Management Science in the World of Today and Tomorrow," *Management Science*, XIII (February, 1967), p. C107.

[34] Herbert A. Simon, *The New Science of Management Decision* (New York: Harper & Row Publishers, 1960), p. 15.

> tific knowledge. Nevertheless, much of our understanding of management science came through operations research, as well as industrial engineering and econometrics. . . . Management science, in its present state of development, has little in the way of general laws and general truths. But from the great body of general management knowledge and experience and from specific operations research applications . . . will come forth fundamental relationships of predictive theory which will distinguish management science as a true science.[35]

The first view, that the two terms, "management science" and "operations research," may be used interchangeably, is the more recent one and is the concept which has been followed in the research for this study.

The techniques of management science include the general area of mathematics, and this may be broken down into the areas of linear programming, queuing theory, the theory of games, inventory models, Monte Carlo techniques, to name a few.[36] In general, the procedures which are employed "can be characterized as the application of scientific methods, techniques and tools to problems involving the operation of systems so as to provide those in control of the operation with optimum solutions to the problems."[37]

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[35]Gifford H. Symonds, "The Institute of Management Science: Progress Report," Management Science, III (January, 1957), pp. 125-129.

[36]Robert M. Trueblood and Richard M. Cyert, Sampling Techniques in Accounting (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957), p. 78.

[37]C. West Churchman, Russell L. Ackoff and E. Leonard Arnoff, Introduction to Operations Research (New York: John Wiley & Sons, Inc., 1957), pp. 8-9.

Many basic sciences such as economics, mathematics, and engineering have been used in the developmental and application stages of management science. The basic procedure of management science is the formulation of a quantitative model depicting all the important interrelationships involved in the problem under consideration and then solving the mathematical model to find an optimal solution.[38] It is particularly in the area of model building that the various sciences are most useful since it is desirable to have the model represent the real world situation as closely as possible.

There are at least three ways in which a relationship between quantitative techniques, such as those of management science, and accounting may exist:

> First, quantitative techniques may be used in performing certain tasks normally associated with accounting. Second, accounting is a prime source of some of the information used to estimate the parameters of various quantitative decision models. And, thirdly, accountants should understand and have access to the decision models used in a firm because some information generated by these models are used in his own tasks or should be included in the information he supplies to decision makers.[39]

Although the concern of this study is with the quantitative aspects of managerial science, there are other branches "which focus on the

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[38] George A. Steiner, Top Management Planning (London: The MacMillan Company, Collier-MacMillan, Limited, 1969), p. 334.

[39] Gerald A. Feltham, "Some Quantitative Approaches to Planning for Multiproduct Production Systems," The Accounting Review, XXXXV (January, 1970), p. 11.

human beings as an individual and as a member of work groups.'[40] These segments will not be explored although the behavioral science implications of the application of the quantitative methods were of concern as far back as the early days of the scientific management movement and currently are gaining in recognition and importance.[41]

The Plan of the Study

This study will begin with developments in standard costing which have been suggested since 1935 although, in some instances, especially when discussing those methods which are in general use, reference may be made to relevant historical background. This will be particularly true in the standards setting discussion.

The subjects to be covered in the next five chapters -- the setting of standards, the analysis of variances, and the allocation of joint costs, make up the major problem areas of standard costing affected by suggested statistical and management science techniques. By discussing

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[40]Elwood S. Buffa, Models for Production and Operations Management (New York: John Wiley & Sons, Inc., 1963), p. 4.

[41]Frederick W. Taylor, The Principles of Scientific Management (Reprint. New York: Harper & Brothers, Publishers, 1942), p. 119: 'There is another type of scientific investigation which has been referred to several times in this paper and which should receive special attention, namely, the accurate study of the motives which influence men.' For a more recent work in this area see, for example: Edwin A. Caplan, Management Accounting and Behavioral Science (Reading, Mass.: Addison-Wesley Publishing Company, 1971) or Frank R. Probst, "Probabilistic Cost Controls: A Behavioral Dimension," The Accounting Review, XXXXVI (January, 1971), pp. 113-118.

each area separately there may be some overlap between areas; this, however, makes for a clearer presentation overall.

Techniques of statistics and management science which will be considered are those applicable to manufacturing cost standards and not those suggested for standards constructed for marketing costs, clerical costs or other costs, although there may be some similarities in the methodology used for the application of standard costs to diverse functional areas. Also, there will be no discussion of any of the behavioral aspects of the several techniques although these may be pertinent, especially with regard to the utility of the procedures for control purposes and performance evaluation. Any control procedure, to be effective, must be understood by those affected by it, and, at times it may be that those affected should also have some voice in establishing the goals to be set for performance (e.g., establishing the control limits). Also, when the results of an operation are used in performance evaluation, the analysis should allow for some failures, particularly when they are brought about by events beyond the control of the person being evaluated.[42]

Chapters II and III consider the impact of statistical techniques upon the setting of standard manufacturing costs. The contributions of scientific management will be considered first, in Chapter II, since these are still widely used, although in a more sophisticated form. Next the

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[42]Caplan, p. 62.

topic of learning curves will be explored because of the ability of such a technique to add a dynamic aspect to the setting of standards. Chapter III examines the need to separate out the fixed and variable components of a mixed overhead cost along with suggested techniques for carrying this out

Chapter IV deals with variance analysis and looks at the meaning of cost control, the utilization of control charts, and the use of various other statistical and control methods, particularly Bayesian decision theory models, multiple regression and multiple correlation models and controlled cost

An extension of variance analysis will be the subject of Chapter V which looks at two linear programming approaches to cost control based on the concept of opportunity cost In addition there will be a discussion of the cost and quantity requirements of the data inputs to linear programming models and the suitability of standard quantities and costs to meet such needs

The topic of cost allocation will be taken up in Chapter VI. Two allocation problems will be considered: coproduct cost allocation and service department cost allocation. In connection with these topics, the use of multiple regression analysis, multiple correlation analysis, matrix algebra and input-output analysis will be considered.

Chapter VII will include, in addition to the summary, some discussion about the possible future trends which may occur, especially in the areas of research on the applicability of various statistical and management science techniques to standard costing

II THE SETTING OF QUANTITY STANDARDS -- IMPACT OF SCIENTIFIC MANAGEMENT MOVEMENT AND LEARNING CURVE ANALYSIS

In order to establish a background against which to measure the impact of the statistical technique on the construction of standards the first section of this chapter will review procedures utilized to determine quantity standards, especially those techniques in use prior to 1945 This will be followed by a brief look at the contributions made by the scientific management movement toward the setting of quantity standards particularly labor time standards. Following this the use of learning curve theory will be presented as a means of eliminating problems created by the absolute standards derived from conventional procedures

Introduction

Standard costs were adopted in the early 1930's as an attempt by management to overcome three major defects in the older types of cost analysis: "the importance attributed to actual costs, the historical aspect of the cost figures, and the high cost of compiling actual costs."[1]

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[1]John G Blocker, Cost Accounting (New York McGraw-Hill Book Company, 1940), p. 552

Other defects of historical costs which were, hopefully, to be eliminated by the use of standard costs were that the actual costs may become known too late to be used for control purposes or that they may be inadequate for purposes of measuring manufacturing efficiency, i.e., they may be atypical.[2] A question at issue, therefore, is whether standard costs, as established in this period -- the 1930's and early 1940's -- actually did eliminate these defects and whether the subsequent use of statistical techniques made for further improvement in the character of standard costs.

Cost accounting texts of the 1930's and 1940's presented price and quantity standards for material and labor costs and price standards for the indirect or overhead costs.

Establishment of Quantity Standards

Although both price and quantity standards are used in variance analysis, the determination of quantity standards for labor will be of primary importance in this chapter because of the early impact of scientific management developments upon them.

> The starting point in the preparation of material quantity and labor time standards is the careful analysis of the engineering specifications, mechanical drawings and lists of parts used in the assembly of the product in question. A knowledge of quantity, type and size of each class of material, or the nature of each labor and machine operation, and of the careful testing

[2]Cecil Gillespie, Accounting Procedures for Standard Costs (New York: The Ronald Press, 1935), p. 2.

of material quantities and the making of time and motion studies is required in determining standard costs.[3]

These basic procedures are presented in less detail, by Harrison (1930), Camman (1932), and Gillespie (1935).[4]

Generally the setting of quantity standards was handled by industrial engineers or experienced shop officials in conjunction with the cost accountant primarily because it was felt that the cost accountant lacked both the practical knowledge and the experience needed to estimate the cost of the work on his own. This delineation of responsibility for the construction of standards was set forth by G. Charter Harrison, who also described the type of standards which the cost accountant, working alone, could be capable of setting:

> Such standards as the accounting division would be in a position to set must necessarily be largely based upon records of past experience, and though data as to past performance are of interest and of value in determining the trend of costs, such data are not suitable for use as standards. . . .[5]

Thus, the introduction of the industrial engineer into the standard setting process had the effect of minimizing the utilization of historical data in the construction of standards.

The above views as to how standards for quantity should be established were reiterated in a more recent work by Henrici.

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[3]Blocker, p. 563.

[4]Camman, p. 7; Gillespie (1935), p. 7; Harrison, miscellaneous pages.

[5]Sowell, p. 225.

> Ideally the standardizing itself precedes the establishing of the standard costs. The supervisors and engineers of the company examine the various jobs and determine how each task should be done, then standardize its performance on the basis of time and motion studies. After this has been done the standard cost accountant translates this standardization into dollars and cents and provides a means for measuring the cost of failure to adhere to it.[6]

Henrici enumerates a number of ways in which standards can be constructed including "an analysis of historical records, simple observation, predetermined time standards and work sampling."[7] These techniques have one characteristic in common -- the standard which is derived is an absolute figure. This characteristic is a major defect in conventional procedures, particularly when coupled with the implied assumption that the unit variable costs remain constant over a wide range of production.[8] These two factors, taken together, act to limit the frequency of the revision of the standard to two circumstances: the noting of "substantial" irregularities and the passage of a "reasonable" length of time from the date of the setting of the standard.[9]

The foregoing pertains mainly to the establishment of material quantity and labor time standards. Standards for overhead expenses

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[6]Henrici, p. 128.

[7]Yezdi K. Bhada, Some Implications of the Experience Factor for Managerial Accounting (Ph.D. Dissertation, University of Florida, 1968), p. 11.

[8]Yezdi K. Bhada, "Dynamic Cost Analysis," Management Accounting LII (July, 1970), p. 11.

[9]Bhada, Some Implications . . . , p. 248.

are more difficult to construct than those for material and labor and are usually handled through budget forecasts.[10] To facilitate the estimation of the standard outlay for each such expense, Gillespie presented a classification of overhead costs into three categories:

1. Fixed charges which are viewed as remaining constant over any volume of production attained "within the usual range of fluctuation",
2. Curved or semi-variable expenses which vary with production but not in direct proportion.
3. Variable expenses which vary directly with production volume.[11]

Despite Gillespie's presentation and the mention of the use of flexible budgets by various authors at least as far back as 1903,[12] no attention is given in the cost accounting texts of the 1930's and early 1940's to the use of an objective technique for the separation of the semi-variable overhead costs into their fixed and variable elements.[13] The methods which were in use, as well as suggested statistical techniques, for the decomposition of the mixed costs will be taken up in the following chapter.

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[10]Blocker, p. 556.

[11]Gillespie (1935), pp. 101-102.

[12]Solomons, p. 48.

[13]In 1947 G. Charter Harrison presented a method which was based on correlation rather than the least squares method that has been suggested for use today. See G. Charter Harrison, "The Arithmetic of Practical Economics" (Madison: Author, 1947), referenced in Upchurch, p. 170.

A number of defects in the use of actual costs for cost analysis were mentioned at the beginning of this section.[14] Utilizing standard costs may eliminate most of these defects, particularly those related to the use of historical cost data. However, the adoption of standard costs has brought with it some new problems, such as the absoluteness of the standards employed and frequently the failure to utilize statistical methodology in connection with semi-variable expenses.[15]

Impact of the Scientific Management Movement

Of the several methods of setting standard costs mentioned by Henrici, the techniques of time study, predetermined time standards, and work sampling may be traced back to concepts emanating from the ideas of the scientific management movement at the beginning of this century. The early quantity standards, particularly labor time standards, can be imputed to F. W. Taylor who believed that it was possible to reduce several aspects of management to an applied science.[16]

> The essential core of scientific management regarded as a philosophy was the idea that human activity could be measured, analyzed, and controlled by techniques analogous to those that had proved successful when applied to physical objects.[17]

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[14]See pages 18-19.

[15]For example, scatter-graphs, regression analysis, correlation.

[16]Buffa, p. 3.

[17]Hugh G. J. Aitken, *Taylorism at Watertown Arsenal* (Cambridge, Mass.: University Press, 1960), p. 16.

The most significant contribution that Taylor and his followers made to the concept of standard costing was the idea that standards of performance could be established for jobs, and then these predetermined standards could be compared with the actual performance times.[18] This was exemplified by the task concept whereby management planned, for at least a day in advance, the work for each worker, specifying what was to be done, how it was to be done and the exact time the job was to take.[19] The establishment of standard processes and standard operating times, which were determined from a careful observation of a "first-class" man carrying out the task, was essential to the development of standard costs.[20] Taylor and his followers "used all the fundamental principles of the modern standard cost system with the exception of the continuous control of cost variances through appropriate cost variance accounts."[21]

In addition to the establishment of labor time standards, Taylor was aware of the existence of the learning process. "No workman can be expected to do a piece of work the first time as fast as he will do it later. It should also be recognized that it takes a certain time for men who have worked at the ordinary slow rate of speed to change to high

[18]Bennett, p. 4 [19]Taylor, p. 39

[20]Solomons, p. 75

[21]Bennett, p. 4

speed.'[22] Although Taylor used an absolute standard time for his wage incentive plan, one based upon the 'quickest time' for each job as performed by a first-class man, he felt that despite all efforts by the workers to remain at the old speed, the faster rate would be gradually approached.[23]

The effective operation of the "Taylor system not only required prompt and accurate reporting of costs; it also generated, as a by-product, data on costs that made quicker and more accurate accounting possible.'[24] The refined cost accounting techniques required to obtain this new information were "based on the routinized collection of elapsed time for each worker and machine in each job, the systematization of stores procedures, purchasing and inventory control.'[25] Because it initiated the idea of using costing as a means of controlling work in process rather than as a technique for recording aggregate past performance, this change in cost accounting acted as a source of managerial improvement.[26]

> The origins of a scientific management approach to management were concerned with the measurement of processes. This was a good start. It gave us management accounting and work study. But the intention to measure things does not exhaust the scientific method, nor does a concern for the processes it commands

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[22]Frederick W. Taylor, Shop Management (New York: Harper & Brothers, 1919), p. 75.

[23]Ibid., p. 59.

[24]Aitken, p. 114.

[25]Ibid., pp. 28-29.

[26]Ibid., p. 18.

exhaust management's role.[27]

Learning Curve Phenomena and Setting Standard Costs

The concept of learning has been ignored in the conventional procedures used for the setting of quantity standards, thus resulting in the development of absolute standards, a defect mentioned at the beginning of the chapter.[28] This section will first present a brief description of traditional learning curve theory followed by a suggested modification entitled "dynamic cost analysis." These will be followed by a discussion of their impact on standard costing and an example of how the traditional approach might be applied to the development of labor time standards.

Traditional Learning Curve Theory

The typical learning curve depicts the relationship between the direct labor hours necessary in the performance of a task and the number of times the operation is performed. The basic theory behind this curve may be expressed as follows:

> . . . a worker learns as he works; and the more often he repeats an operation, the more efficient he becomes with the result that direct labor input per unit declines . . . The rate of improvement is regular enough to be predictable.[29]

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[27]Stafford Beer, Management Science (Garden City, N. J.: Doubleday & Company, Inc., 1968), pp. 26-27.

[28]See page 18.

[29]Frank J. Andress, "The Learning Curve as a Production Tool,"

The curve that is developed from the data is based upon the number of trials involved, not time per se [30]

The curve from which the rate of improvement may be determined results from the plotting of the direct labor hours-output or direct labor cost-output data which are obtained for a given operation. These figures may be from historical data developed from the performance of similar operations or, if such data are not available, there are tables which may be used [31] To make the prediction of time, or cost, necessary to produce a given output, the data are plotted on log-log graph paper which will produce a linear relationship between the variables Figures 1 and 2 show some typically shaped curves The learning

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Harvard Business Review, XXXII (January-February, 1954), p 87 An extended illustration of the operation of the theory is given by Crowningshield, p 147 'The pattern which has been derived from statistical studies can be stated as follows each time cumulative quantities are doubled, the cumulative average hours per unit will be reduced by some constant percentage ranging between 10 and 40 per cent, with reductions of 40 per cent extremely rare '

"Various 'rates of learning' have achieved some recognition as appropriate to various types of manufacture, such as assembly (70-80%), machining (90-95%) welding (80-90%), and so on " E B Cochran, "New Concepts of the Learning Curve " The Journal of Industrial Engineering, XI (July-August, 1960), p 318

[30] Patrick Conley, "Experience Curves as a Planning Tool," IEEE Spectrum (June, 1970), p 64

[31] One such table is 'The Standard Purdue Learning Tableau (for expected efficiency in percent for inexperienced workers) ' Efraim Turban, "Incentives During Learning -- An Application of the Learning Curve Theory and a Survey of Other Methods," The Journal of Industrial Engineering, XIX (December, 1968), p 601

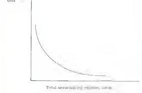

Figure 1 Learning Curve as Plotted on Regular Graph Paper (Linear Scale)

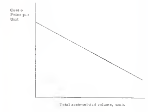

Figure 2 Learning Curve As Plotted on
Log-Log Paper

process, despite the continuous downward slope shown on the log-log scale (Figure 2), slows down to a point where it appears to be static when displayed on linear-scale graph paper (Figure 1). This phenomenon occurs because the curve is based on a relationship to trials rather than time.[32]

The opportunity for learning exists mainly in operations which present a chance for efficiency improvement. Such processes generally will not be routine or repetitive, nor will they be machine-paced. The greatest occasion for learning occurs in those tasks which are complex and lengthy and produce a limited number of units requiring "a high degree of technical skill," e.g., the manufacture of aircraft.[33] The possibility of learning is also negligible on operations which have been performed for some time. This is evident when the learning curve is plotted on linear graph paper and both the initial decline and the later flattening out of the curve may be seen (see Figure 1).[34]

> The hypothesis that experience promotes efficiencies which lead to a decline in cost with increased production is still acceptable, but it is dangerous to generalize that such declines take place by means of a constant percentage whenever quantities produced are doubled.[35]

The learning curve, as traditionally determined, may be affected by

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[32]Conley, p. 64

[33]Crowningshield, p. 149

[34]Winfred B. Hirschmann, "Profit From the Learning Curve," <u>Harvard Business Review</u>, XXXXII (January-February, 1964), p. 125.

[35]Bhada, "Dynamic Cost Analysis," p. 14.

several factors which are not normally considered.[36] These factors, some of which will be discussed below, may change the basic shape of the curve so that the linearity assumption will be subject to question.[37]

<u>Dynamic Cost Analysis -- A Variation of Application of Learning Curve Phenomenon</u>

This is an approach to learning curve theory developed by Bhada which considers the possibility of a nonlinear relationship of learning to activity.[38] The term "experience" is used by Bhada rather than "learning" because interest centers on "the phenomenon of gaining positive efficiency, observable in the form of quantitative improvement in the course of an operation being repeated over a period of time" by a group or organization rather than with "the acquisition of knowledge on the part of an individual" -- learning.[39]

The dynamic cost function is developed from production data which Bhada defines as "manufacturing information collected from continuous operations."[40] This function is composed of a number of elements and sub-elements each of which may have a different rate of improvement. Two examples of this are: 1) the unit cost function which normally is

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[36]See Samuel L. Young, "Misapplications of the Learning Curve Concept," <u>The Journal of Industrial Engineering</u>, XVII (August, 1966), pp. 412-413, for a discussion of typical factors.

[37]Bhada, "Dynamic Cost Analysis," p. 14. [38]<u>Ibid.</u>, p. 11.

[39]Bhada, <u>Some Implications</u> . . ., pp. 22-23. [40]<u>Ibid.</u>, p. 25.

as aggregation of several types of cost such as material, labor and overhead; and 2) the direct labor hour curve which may be made up of assembly time, sub-assembly time, and parts manufacturing hours.[41] Since, in either instance, each cost element may be affected by a different rate of improvement because of the continuous production factor, the dynamic cost function, which is a summation, will not necessarily be linear.[42] (See Figure 3, Curve A', for example.)

The dynamic function may be affected by two determinants; the first of these is the exponent of the slope of the experience curve which is influenced by "during-production" changes and improvements. The second is the labor hours which are established for unit one and which are determined by "pre-production" factors.[43] These latter determi-

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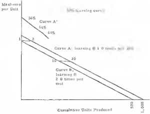
[41] Ibid., p. 263.　　[42] Ibid.

[43] Bhada, "Dynamic Cost Analysis," p. 12. The "during-production" and "pre-production" factors are defined as follows:

1) "Decisions made regarding the anticipated volume of production can substantially affect the experience rate. The anticipated volume of production can have as its two components expected rate of production and the estimated length of production which can conceivably influence engineering and production decisions, which in turn can affect the experience rate." Bhada, Some Implications . . . , p. 177.
2) "Once pre-production planning is completed and the product put into production, the process of further improvement starts. In spite of considerable care taken at the pre-production stage, there are bound to be coordination errors and peculiarities which can be improved upon in the process of production. Thus tooling can be bettered, engineering updated, production methods and scheduling improved, and better coordination achieved as the particular deficiencies are evidenced. Above all, the factor of human labor being introduced presents opportunities for learning and improvement with increased production." Ibid., pp. 180-181.



Source Cochran p. 319

Figure 3 Various Examples of Learning Curves -- Log-Log Scale

nants, which are reflected in the experience curve, were outlined by Hall in 1957.[44]

Additionally, the dynamic function can be affected by design changes which may have a substantial impact on the cost of complicated products. Two factors are responsible for the effect: the extra cost which is incurred to introduce the changes and the higher labor costs arising because of the increased labor hours necessitated by the loss of experience.[45] The increased costs should be reflected in the labor standards as well as in the estimated price of the product. There also is the possibility that the reduction trend existing before the design change will no longer exist after the initial impact of the change has worked off thus necessitating a new experience curve with a different rate of improvement.[46] This, too, should be reflected in the product labor standard.

The primary difference between dynamic cost analysis and the traditional learning curve is that the former keeps open the possibility of nonlinear curves (as plotted on log-log paper). Secondly, dynamic cost analysis adjusts for the effects of some technological change through the "during-production" changes, whereas the traditional procedure considers technology as remaining completely fixed during the time a given learning curve is felt to be operational. A final difference between

[44]Lowell H. Hall, "Experience with Experience Curves for Aircraft Design Changes," N.A.A. Bulletin, XXXIX (December, 1957), p. 59.

[45]*Ibid.*, p. 60. [46]Bhada, *Some Implications*, p. 254.

the two concepts is that dynamic cost analysis is more interested in the group whereas learning curves in the traditional sense tend to look at individual performances.[47] The concept of variable costs in both approaches differs from the traditional definition of such costs. Customarily the variable cost per unit is felt to be constant, but the "dynamic cost quantity relationship indicates [that] variable cost per unit tends to decline with increased production" in a fashion analogous to that of unitized fixed costs.[48]

Impact on Standard Costs

When learning curve theory is utilized in conjunction with the development of standard costs, some of the defects caused by absolute standards may be overcome. Because it is capable of predicting changes, the traditional learning curve is useful in the establishment of standards of performance.[49] It is especially helpful in setting time standards in the early stages of a productive operation which, when tied in with a wage incentive system, may act to increase productivity.[50] If a

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[47] Ibid., pp. 22-23.

[48] Yezdi K. Bhada, "Dynamic Relationships for Accounting Analysis," Management Accounting, LIII (April, 1972), p. 55.

[49] Lloyd Seaton, Jr., "Standard Cost Developments and Applications," Management Accounting, LII (July, 1970), p. 66.

[50] Turban, p. 600. This article presents a description of how one company set up an incentive system while using learning curves.

learning phenomenon were recognized, but the conventional procedures of setting standards were followed, it would be necessary, although highly impractical, to calculate a new labor standard by means of engineering studies, etc. for each unit produced. By incorporating the learning curve concepts into the calculation, a "progressive and systematic" standard can be developed which automatically yields a new, lower value for each unit produced and such values may be determined in advance.[51] Such a standard provides a more viable reference point when it is being taken into consideration for cost control and part of the variance analysis and performance evaluation can be an analysis of the individual's, or the group's, rate of learning as compared to the expected rate.

An additional advantage evolving from a consideration of learning rates is the possibility of more frequent revisions of the standards. This

[51]Ralfe Wyer, "Learning Curve Techniques for Direct Labor Management," N.A.A. Bulletin, XXXX (July 1958), p. 19. Bhada, in Some Implications . . . , pp. 251-253, indicates a number of ways in which the effects of learning may be brought into the standards, primarily by means of a sliding scale or an index, and also discusses a number of cases where one would, or would not consider the effects of learning.

The factors such as tooling, supervision, parts design -- the "during-production" changes -- can be included in the rate of improvement by the following steps:

"1. Identify the relative importance of each factor to the learning rate.
2. Establish the influence of the particular factor upon the unit at which standard cost will be achieved (in effect the rate of learning).
3. Work out a statistical combination of each factor to permit computing the overall rate of learning."

Cochran, p. 320.

possibility acts to eliminate one of the major defects of conventional technique for setting standards — infrequent revision. An illustration, in the following section, provides an example of how the learning process may be incorporated into standard costing.[52]

Although the learning curve is generally considered in the estimation of labor hours, it also affects labor costs "costs go down by a fixed percentage each time the number of units doubles."[53] The technique may be applied effectively to all manufacturing costs which can show a direct proportional association to labor hours or cost. Such costs are often expressed as $x per direct labor hour. Thus, as direct labor hours per unit decrease with experience, so do these indirect costs, and the reduction is particularly dramatic in the early stages of production.[54] The costs to which the learning curve concept cannot be applied are those which decrease at a nonconstant rate, such as material costs, or those fixed costs which are related to the provision of capacity.[55] However, although there is no direct relationship which can be displayed between learning and material costs, several indirect effects are possible because with learning comes increased efficiency which would lead to a more effective use of the raw materials.[56] Such a possibility should

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[52]See pages 40-46.

[53]Conley, p. 64.

[54]Crowningshield, p. 150.

[55]*Ibid.*

[56]Bhada, *Some Implications*, pp. 194-195. Bhada notes that "total material cost could be influenced by the quantity of raw material used, the varieties of components involved, the quality of the materials

be taken into consideration if possible, when setting up the material quantity and material price standards.

Examples of the Application of Learning Curves to Standard Setting

Two approaches have been suggested for a learning curve analysis of cost, each one using a different reference point in the learning curve as the starting point. The first of these employs unit one as the reference or standard; the second, some future predetermined unit X which represents the point of standard company or industry experience.[57] Because of inefficiencies existing at the beginning of a productive operation, it is felt to be more appropriate to choose the latter method -- that is, a reference point occurring somewhere further in the production run, e. g., after the first lot is produced. The use of a future reference point also resembles the concept expressed by F. W. Taylor when he established a "quickest time" toward which all other workers were to strive and which then acted as a standard. In either procedure, the standard time will continue to be developed by means of time studies or other engineering methods which then are correlated with the reference point. The use of such a correlation procedure helps to increase

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and the price at which these ingredients were acquired."

[57]Cochran, p. 319.

the reliability of the results.[58]

When the future reference point method is used, it must be remembered that "any change in learning rate is equivalent to a change in the unit at which the standard cost is reached," and thus, in turn, shifts the cost curve.[59] For example, see Figure 3 on page 33; curve A uses the cost of 1,000 as the standard cost, but curve B, which doubles the learning rate, reaches the standard cost at unit 500. Because of this phenomenon, the importance of the determination of the appropriate learning rate becomes apparent when it is to be used in forecasting and controlling costs.[60] Appendix A presents a diagram which indicates a procedure for estimating hours in those situations in which a learning curve is to be employed.

An essential step in the procedure is the analysis of actual experience "in order to determine at what point in the unit sequence the standard used will be achieved."[61] When this is done, the learning curve needs to be set up only for the number of units required to reach the standard cost.[62]

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[58]E. B. Cochran, Planning Production Costs: Using the Improvement Curve (San Francisco: Chandler Publishing Company, 1968), p. 203; Robert Boyce Sweeney, An Inquiry into the Use of Mathematical Models to Facilitate the Analysis and Interpretation of Cost Data (Ph.D. Dissertation, The University of Texas at Austin, 1960), pp. 397-398.

[59]Cochran, "New Concepts . . . ," p. 319. [60]Ibid.

[61]Cochran, Planning Production Costs . . . , p. 257. [62]Ibid.

For example, supposing that a company fabrication department on a 90 per cent slope finds that a given product reaches a cost of 150 hours at unit 300 while its standard indicates a cost of only 75 hours. We can immediately calculate that the 75 hour [standard] cost would be reached at unit 28,700 [by means of appropriate formulae or tables], even if the company never produced any such number of units to prove this point.[63]

<u>Extended illustration of the use of learning curves in setting or adjusting labor standards</u>[64]

In the submission of a contract bid, an initial step is the development of the cumulative average time per unit required in the manufacturing of the entire lot. This estimate generally will differ from the standard hours. The expected hours may be computed by any of several techniques, e.g., mathematical models, logarithmic graphs or conversion factors.[65] These data are then used in the interpretation of the labor efficiency reports. "The projected hours in the learning curve may be used to adjust standards each month or merely as a supplemental device for effective control of labor costs."[66]

To illustrate the foregoing, assume the firm receives an order for 2,000 items, the production of which is to be spread over a period of

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[63] Ibid.

[64] Sweeney, pp. 398-407. The example being presented is summarized from one presented by Sweeney with some simplifying alterations in the descriptions and tables.

[65] Ibid., mathematical models, pp. 325-352; logarithmic graphs, pp. 352-365; conversion factors, pp. 366-373.

[66] Ibid., pp. 368.

twelve months. Two departments will be required in the total operation with the following relevant data:

	Cumulative Average Hours	Standard Hours	Learning Curve	Lead Time
Department A	30	32	90%	2 months
Department B	70	69	78%	1/2 month
	100	101		

The production and shipping schedules which must be met are presented in Table 1. These data may be used in the derivation of a series of standards ("realistic estimates") as follows:

1. "compute the total labor hours expected to be incurred each month as well as the average labor hours per unit each month",[67] these figures are presented in Table 2.
2. Compare actual hours to the estimated hours as a technique of controlling labor efficiency as shown in Table 3.

"Column 4 of [Table 3] indicates the efficiency which can be expected if standard hours are not adjusted in accordance with hours projected using the learning curve."[68] As long as the actual efficiency equals or exceeds the projected, performance is felt to be satisfactory. Thus, the desired efficiency target is to produce in accordance with the projected hours, and the use of less than projected hours leads to efficiency levels which exceed 100 percent. The use of the constant standard time (column 3, Table 3) produces excessive unfavorable variances for approximately half of the production period and favorable

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[67] Ibid., p. 408.   [68] Ibid., p. 406.

Table 1

Production and Shipping Schedule

	Production				Shipping	
	Department A		Department B		units shipped	
Month	per month	cumulative	per month	cumulative	per month	cumulative
1	25	25				
2	75	100	12	12	--	--
3	150	250	50	62	25	25
4	250	500	113	175	75	100
5	250	750	200	375	150	250
6	250	1 000	250	625	250	500
7	250	1 250	250	875	250	750
8	250	1,500	250	1,125	250	1 000
9	250	1 750	250	1,375	250	1,250
10	250	2 000	250	1,625	250	1,500
11		2,000	250	1,875	250	1 750
12		2,000	125	2,000	250	2,000

Source: adapted from Sweeney, p. 401

The following equations [illegible]*

unit 1: $t_1 = A_{c_X} X^k$

second month $A_{c_{X_{a-b}}} = \frac{t_1}{X_a - X_b\ 1}\ X_b^{(1-k)} - (X_a - 1)^{(1-k)}$

Total hours for month $T_{X_{a-b}} = A_{c_{X_{a-b}}} (X_b - X_a \quad 1)$

where the following meanings are attached to the variables

X any unit number

t_1 the time (or cost) for any individual unit X

T_X the cumulative total time (or cost) through any unit X

A_{c_X} cumulative average time (or cost) for any unit X

k slope (tangent) of learning curve

X_a unit number a

X_b unit number b

The formulas above for $A_{c_{X_{a-b}}}$ and $T_{X_{a-b}}$ are dealing with averages for a specific lot of production, where x_a is the first unit of the lot and x_b the last unit

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*Sweeney, p. 333

Table 2

Expected Labor Hours by Mon[illegible]
During Progress of Contract

	Department A		Department B		Grand
Month	per unit	Total	per unit	Total	Total Hours
1	58.4	1,460	[illegible]	[illegible]	1,460
2	43.6	3,270	438.1	5,257	8,527
3	37.1	5,558	196.5	9,824	15,382
4	32.9	8,230	126.1	14,256	22,486
5	30.6	7,659	92.5	18,506	26,165
6	28.6	7,156	74.2	18,549	25,705
7	27.8	6,943	63.9	15,986	22,929
8	26.9	6,735	57.7	14,416	21,151
9	26.3	6,566	53.2	13,295	19,861
10	25.7	6,423	49.8	12,451	18,874
11			47.2	11,789	11,789
12			45.4	5,671	5,671
		60,000		140,000	200,000

Source: adapted from Sweeney, p. 402

Table 3

Projected Labor Efficiency for Contract Period

Month	Total Projected Hours[a]	Total Standard Hours[b]	Projected Efficiency %[c]
1	1,460	800	54.8
2	8,527	3,228	37.8
3	15,382	8,250	53.6
4	22,486	15,797	70.3
5	26,165	21,800	83.3
6	25,705	25,250	98.2
7	22,929	25,250	110.1
8	21,151	25,250	119.4
9	19,861	25,250	127.1
10	18,874	25,250	133.8
11	11,789	17,250	146.3
12	5,671	8,625	152.1

Source: Sweeney, p. 405.

[a]column 6 of Table 2

[b]32(x) + 69(y) where x is the monthly unit production of department A and y, the monthly unit production of department B, from Table 1

[c]projected efficiency % = total standard hours/total projected hours

variances to [illegible] directly attributable to learning

The use of projected hours as the "standard time" would give management a better base against which to measure performance Any variance which still occurs most likely will be caused by other factors, e g , machine downtime If the firm were operating at the point where traditional total standard hours exceeds the total projected hours, the traditional variance analysis technique probably would show a favorable variance if the only factor causing the difference was learning However, the magnitude of the favorable traditional variance could be increased, reduced or eliminated if other factors either favorable or off-setting, were also influencing labor hours.

Also, if both sets of figures are available, as shown in Table 3 columns 2 and 3, and an additional column were to be added which shows the actual hours worked each month an actual efficiency could be calculated and compared with the projected (column 4 of the table) to see if the learning is progressing as expected This would tend to give management another control factor -- if the actual efficiency differs significantly from the projected possibly the estimated rate of learning is in error

Summary

After a brief statement concerning the state of the art of setting manufacturing standards, two topics were considered scientific man-

agement and learning curve phenomena. The scientific management movement provided the concepts behind the time and motion studies which were initially used to determine quantity standards for labor in particular. Scientific management and the traditional methods of estimating standards represented static procedures in that a standard was set up as of a particular date and then revised at regular intervals. The use of learning curve phenomena represents a more dynamic method of determining labor time standards. In certain situations the labor time taken per unit (and consequently the cost) declines according to the predicted effects of learning, eventually attaining the desired standard time. The actual rate of decline may be compared to the predicted rate to see if the standard time is being approached as anticipated.

A question was posed at the beginning of the chapter regarding the effectiveness of statistical techniques in enabling standard cost systems to overcome the defects apparent in the early historical cost system.[69] The learning curve and its variant, dynamic cost analysis, are both procedures to keep certain standards timely. The revisions are predictable and almost automatic. With learning curve information, the cost accountant is able to establish what the labor time will be, and therefore the costs, without excessive effort.

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[69]See page 19.

III IMPACT OF DEVELOPMENTS AFFECTING THE ANALYSIS AND STANDARDIZATION OF MIXED COSTS

This chapter will first examine some of the traditional techniques which have been, and still are, in use for the decomposition of mixed costs into their two cost components. This will be followed by a discussion of statistical techniques which have been suggested as a solution to the separation problem and their impact upon the setting of standard costs

Introduction

Standards are established for three main groups of manufacturing costs direct materials direct labor and overhead There rarely is any problem in determining the fixed and variable elements of the first two cost categories This is not the case, however, with overhead which represents a blanket category covering many types of costs, some clearly fixed or variable in nature and others showing neither clearly defined fixed or variable characteristics The separation of these mixed overhead costs into their fixed and variable components is necessary for a clear-cut displayal of product cost standards and subsequent use in cost and variance analyses, flexible budgeting and direct standard costing There also is a need to know the variable costs for

the linear programming models, as will be discussed in Chapter V. The separation must be done as carefully as possible since any measurement errors occurring in the process will affect the evaluation of the performance of those who exercise control over such costs.[1]

Definitions

Variable costs are commonly thought of as those which tend to fluctuate in total amount with changes in output. For a variety of computational purposes these are computed to be constant per unit of output. In contrast, fixed costs are defined as those which tend to remain constant over wide ranges of output, but vary in an inverse relationship on a per unit basis. Another way of viewing these cost categories is that variable costs are those which are related to operational activity within an existing firm, and fixed costs are those related to the establishment of both the physical and managerial capacity of the business.[2]

These concepts of fixed and variable represent two extremes of cost behavior and aid in the categorization of a number of costs, e.g., material and labor used directly in the production of the product, executives' salaries, property taxes. In between the extremes there are many costs which contain elements of both fixed and variable costs,

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[1]Dopuch and Birnberg, p. 352.

[2]Separating and Using Costs as Fixed and Variable, Accounting Practice Report No. 10 (New York: National Association of Accountants, June, 1960), p. 6.

e g , an expense which is made up of a flat fee plus a per unit charge These costs generally are referred to as semi-variable or mixed costs [3] Another type of costs which causes difficulty for the analyst is the step-like costs which are defined as those costs which "change abruptly at certain output levels."[4] These costs may be almost variable in nature "step-variable," when changes in their amounts can occur with small increases in output e g., situations where a new foreman is needed every time an additional fifty men are hired,[5] or alternatively, semi-fixed, where the changes may be less frequent to the extent that they may be safely ignored within the relevant range of production [6]

Traditional Separation Methods

Accountants have been fascinated by the problem of how to separate fixed and variable costs for more than half a century [7] The need to carry out such a process was given emphasis with the development of

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[3] These definitions closely resemble those presented by Gillespie as stated in the introduction to Chapter II, p. 22

[4] Charles Weber The Evolution of Direct Costing Monograph 3 Center for International Education and Research in Accounting (Urbana, Ill The University of Illinois, 1966), p. 7

[5] The handling of semi-variable step-costs will not be taken up explicitly by any of the procedures to be mentioned in this chapter If the steps are small enough the costs may be treated as variable (Dopuch and Birnberg p 14) If the steps are larger as in the example cited above a schedule could be set up showing the changes in variable cost at the appropriate outputs

[6] Horngren, p 24

[7] C Weber, p 16

flexible budgeting and various related techniques, e.g., direct costing, direct standard costing.[8]

Although cost accounting texts of the 1930's and early 1940's recognized the necessity for a splitting of mixed costs into their fixed and variable components, they often did not suggest a technique for carrying out the separation process. At least two methods did exist during this period, however, and both were discussed in the periodical literature and used in practice.[9] Neither of these was statistical in nature, nor did they fall under any of the management science technique classifications.

One of these methods is called the "accounting approach." This technique studies the firm's chart of accounts and classifies all costs contained therein into three categories: fixed, variable and semi-variable; then the semi-variable costs are reclassified into the two main categories on the basis of a subjective, arbitrary decision as to whether the cost is predominantly fixed or variable.[10] No one cost item is divided into the two components: a cost is either considered mainly fixed or mostly variable. Because of the simplicity of this procedure, its use was strongly advocated by Joel Dean in 1952.[11]

Another of the more traditional separation processes is the "high-low" approach which looks at several sets of data in order to establish

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[8] Ibid.

[9] Ibid., pp. 17-22.

[10] Ibid., p. 7.

[11] Ibid., p. 21.

either a cost-output or cost-input relationship. The first step of the procedure is to determine the difference between the total costs at the upper and lower bound of the independent variable (input, output). This difference, total cost at highest volume less total cost at lowest volume, which must always be positive, is then divided by the corresponding range of the independent variable.[12]

> For many writers, this calculation leads to the average variable costs and allows [for] the determination of the total amount of the fixed costs as well as the total costs connected with any intermediate level of the independent variable.[13]

Both the accounting approach and the high-low approach procedures suffer from serious deficiencies. In the case of the accounting approach, there is a tendency to maintain the initially determined fixed and variable labels for the costs, even if their behavior changes over time.[14] The technique fails to recognize that costs classified as fixed in the immediately past period, for example, may now be semi-variable.[15] The high-low procedure may be affected by two shortcomings. "First,

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[12] Dopuch and Birnberg, pp. 52-53. The method of calculation may be seen from the following example:

$$VC/unit = \frac{TC_{highest} - TC_{lowest}}{x_{highest} - x_{lowest}}, \quad x = output$$

$$FC = TC_{highest} - VC(x_{highest}) = TC_{lowest} - VC(x_{lowest})$$

$$VC/unit = (\$51,000 - \$42,000)/(4,000 - 3,000) = \$9/unit$$

$$FC = \$51,000 - \$9(4,000) = \$42,000 - \$9(3,000) = \$15,000$$

[13] C. Weber, pp. 6-7.

[14] *Ibid.*, p. 7.

[15] *Ibid.*, pp. 21-22.

it may result in negative fixed costs[16], the occurrence of negative fixed costs does not, by itself create any problem except that they may arise solely through the mathematical formula used and not from actual circumstances.[16] Second, it fails to consider carefully those semi-variable costs which move in a step fashion.[17]

Statistical Analysis

Cost functions may be estimated more rigorously by means of statistical curve fitting. The use of statistical methods to carry out the separation process is not a new concept, but is an approach traceable to the work of Joel Dean (1936).[18] Statistical curve fitting is a term which encompasses a group of techniques used to investigate individual relationships which may, or may not be linear or require the analysis of several variables.[19] "Statistical techniques applied to studies of cost behavior lead to more scientific analyses of cost variation with volume, particularly if factors other than volume are influencing cost behavior."[20]

Statistical approaches which are most commonly used in the sepa-

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[16] *Ibid.*, p. 7. To see how negative fixed costs could arise, change the output figures in the example in footnote 12 to 6,000 and 5,000 units respectively. The VC/unit will remain $9, but FC = -$3,000.

[17] *Ibid.*

[18] *Ibid.*, p. 22.

[19] Dopuch and Birnberg, p. 53.

[20] Crowningshield, p. 481.

ration of fixed and variable costs are based upon the scatter graph method and the least-squares techniques.[21] These procedures are independent of all other techniques and are especially helpful in making preliminary studies. Their usefulness for detailed studies is limited however, because of their ability to deal with only a relatively small number of aggregated cost groups in the investigation, particularly if simple linear regression is being used.[22]

> The tools (i.e., scatter charts or method of least-squares, etc.) are used to discover the presence of a fixed element in a cost and to disclose its size and the variability of the remainder, all in terms of operating volumes of the present or immediate past or future. The employment of the tools requires correlation of volume in physical terms, such as units produced or labor hours, with cost in dollars for items or groups of items.[23]

The fixed and variable components of the semi-variable overhead costs should be determined before product standard costs are computed. This separation must be done in order to determine the two overhead rates, fixed and variable, each of which is then dealt with in a separate cost category with different techniques of analysis. If there is any measurement error in this separation procedure, it will affect the evaluation of the performance of those who have control over the costs.[24]

Variable costs generally are related to some activity or volume base. Typically some base expressive of productive activity is chosen

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[21]C. Weber, p. 22.   [22]Ibid., p. 7.

[23]Separating and Using Costs as Fixed and Variable, p. 8.

[24]Deputh and Birnberg, p. 352.

as the independent variable (e. g., direct labor hours, output volume), but very little guidance is given in the literature as to how to select the appropriate base.[25] The inaccurate choice of a base, one with an insufficient relationship to the cost being analyzed, may render ineffective the decision arrived at, regardless of the choice of separation procedure.[26] If the base which has been chosen is incorrect for a particular cost, it could result in the improper charging of the cost elements to the various departments. To some extent, however, the scatter-graph and least-squares analysis may be used to overcome this problem, as will be discussed later.[27]

Graphical Statistical Analysis

The scatter-graph is a graphical display of the cost behavior pattern as related to the chosen independent variable. It plots the various cost-volume pairs of the sample being analyzed. While the procedure of the graph is not as precise as the least-squares method, there is a built-in

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[25]The most explicit statement of a set of criteria to be used in selecting a volume base may be found in Horngren, pp. 230-231. These criteria are:

"1. Cause of Cost Fluctuation . . .
2. Independence of Activity Unit . . .
3. Ease of Understanding . . .
4. Adequacy of Control over Base . . ."

Crowningshield, pp. 78-79, and Shillinglaw, pp. 408-409, mention the first and third of the above criteria.

[26]R. S. Gynther, "Improving Separation of Fixed and Variable Expenses," N.A.A. Bulletin, XXXXIV (June, 1963), p. 30.

[27]See pages 56, 64-66.

measure of reliability in the technique: the degree of correlation between the cost and volume is apparent when the observations are plotted.[28] (For example: Are the points bunched together? Do they display a definite trend? Are they widely dispersed over the entire graph?) The graph may also highlight any erratic cost behavior which might have developed after the apparent relationship has been established.[29] (For example: Is there some dramatic deviation of the points from the earlier pattern?) The plotted cost-volume observations are given meaning insofar as their ability to designate the amount of fixed cost and the degree of variability of the balance of the cost, by the position of a curve which may be fitted to the points either by inspection or from a mathematical formula.[30]

The visual inspection method of fitting the curve is the simplest procedure in that it requires neither formulas nor calculations, only the experience of the person carrying out the process, but it has one serious limitation. The use of inspection introduces a subjective element into the results which may be removed by fitting the line mathematically.[31] This technique, however, may be used satisfactorily as a basis for further, more rigorous investigation and analysis.[32]

The accounting approach (as described on page 51) may be made

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[28] Crowningshield, p. 483 [29] *Ibid.*

[30] *Separating and Using Costs as Fixed and Variable*, p. 12.

[31] Crowningshield, p. 483 [32] C. Weber, p. 8

more precise and more objective by supplementing it with a graphical statistical analysis Such an analysis would involve the setting up of a scatter-chart of the cost-output observations and visually fitting a curve to the data [33]

Regression Analysis

The mathematical procedure used to eliminate the personal bias is regression analysis [34] Under this general heading fall various techniques ranging from least-squares analysis, or simple linear regression, which deals with only two variables, one independent and one dependent through multiple regression which looks at the effect of several independent variables on the single dependent variable, to curvilinear situations which deal with the nonlinear problems The curvilinear models can be changed to one of the two types of linear models through the use of logarithms and, thus will not be discussed separately

Simple linear regression

Inasmuch as it is generally believed that each overhead cost is related primarily to only one independent variable the method of simple linear regression least-squares analysis in the separation procedure most likely to be used once a rigorous statistical approach is decided upon This is the least complicated of the regression techniques and will result in an objective mathematically precise separation of the

[33]Ibid , p 22 [34]Crowningshield, p 485

[illegible] variable co[illegible] [illegible]r two components [35]

Least-squares analysis, when used to calculate cost standards will give an estimate of the behavior of each cost in relation to its output measure. The accuracy of the estimate, thus derived, will increase with the number of cost-output (cost-input) observations obtained within a homogeneous period of time.[36] Simple linear regression often is presented along with a scatter-graph, in order to show its ability to fit the trend line, but the existence of a graph is not a necessary part of the analysis of a cost into its components.

Multiple regression

It is very difficult to ascertain if the traditional separation processes, especially those using output as the independent variable, provide valid results and that the variable cost component, thus derived, varies its relationship to output from one period to the next.[37] These methods also do not tell if an average variable cost which might be calculated from several of the period costs is useful for any of the several uses of variable costs such as to provide linear programming coefficients

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[35]Batty, p. 228.

[36]Myron J. Gordon, "Cost Allocations and the Design of Accounting Systems for Control," In Readings in Cost Accounting Budgeting and Control, Ed. Wm. E. Thomas, Jr. (3rd ed.; Chicago: South Western Publishing Co., 1968), p. 580.

[37]George J. Benston, "Multiple Regression Analysis of Cost Behavior," The Accounting Review, XXXXI (October, 1966), p. 658.

or data for flexible budgeting.[38] Least squares analysis, while an improvement over the traditional techniques and a handy expedient prior to the widespread availability of computers, is only able to look at the effects of one variable on cost.[39] The move to multiple regression makes possible the estimation of the effect upon overhead costs of various cost-causing factors. "it measures the cost of a change in one variable, say output, while holding the effects on cost of other variables constant."[40] In this way it may be possible to establish a more comprehensive basis upon which to set the standard overhead rate because some factor which might have a definitive affect upon the level of the cost may be taken into consideration, and other factors which may have an effect but are uncontrollable, e.g., the weather, may be eliminated from the model.[41] The determination of the type of cost estimate is useful for many function, including the preparation of flexible budgets, which "take account of changes in operating conditions."[42]

Whether or not it is feasible to use multiple regression in a parti

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[38] Ibid. [39] Ibid. [40] Ibid.

[41] Ibid. Benston gives an example of such factors in terms of a shipping department. The main factor affecting shipping costs would be the number of orders processed, but the weight of the packages is an additional factor which might be considered -- it costs more to ship a heavy package than a light one -- and the weather, an uncontrollable factor which may also affect delivery cost -- bad weather slows delivery time and thus increases cost -- is a factor which might be eliminated from the analysis, if possible.

[42] Ibid.

cular situation should be based upon the results of comparing the "marginal cost of the information" to the "marginal revenue gained from it."[43] Multiple regression analysis is especially helpful when used to estimate fixed and variable cost components to be employed in recurring decisions and the preparation of production overhead standards fits into this area.[44] Recurring problems normally relate to repetitive situations which require schedules depicting _expected_ costs and activity."[45] Because of the frequency of the occurrence of the problem, the situation is most likely to be one in which the marginal cost of obtaining the data each time they are needed would exceed the marginal revenue received from the data. Multiple regression analysis techniques will provide, for example, an estimated marginal cost of a unit change in output with the total cost of other relevant factors accounted for, which may then be applied to several specific decisions involving that operation, any of which may also be part of standard costing, e.g., flexible budgeting, variance analysis, inventory costing, or pricing.[46] One-time problems would not benefit from the use of multiple regression for cost estimation, just as they probably would not be involved with standard costs, since these normally occur infrequently and require explicit consideration of the particular circumstances existing when the decision is to be made.[47]

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[43] _Ibid._, p. 659 [44] _Ibid._, p. 660

[45] _Ibid._ [46] _Ibid._ [47] _Ibid._

Difficulties in applying regression analysis

The line which is derived from the least-squares analysis represents the best fit for the data. However, "adapting it for use in determining cost behavior must be approached with care. This is because of a phenomenon known as drift and concerns some of the points used in the calculation."[48] Because of the tendency of costs "to drift upward over time . . . statisticians refer to the straight line established by using the least squares method as the trend line. It develops a trend but it may not be representative of the status [of the cost] at any given point of time."[49]

Another difficulty with regression analysis concerns the ability of least-squares analysis to fit a straight line to any set of cost data, regardless of the cost behavior pattern exhibited by the points on the scatter-graph.[50] Thus, a line may be fitted to data which are highly erratic or which, while not erratic, bear no true relationship to each other. The reliability of the results obtained from a least-squares analysis is dependent upon the assumptions used regarding the basic structure of the cost curve: "the adequacy of an assumed linear and homogeneous function might be very difficult to prove and hard to maintain for practical purposes."[51]

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[48]Li, pp. 602-603. [49]Ibid., pp. 603-604.

[50]Crowningshield, p. 485.

[51]C. Weber, p. 8.

A third shortcoming of the statistical techniques discussed above — scatter graphs and regression analysis — is that "they are only concerned with the past which may be marked by conditions that will not pertain to the future."[52] Historical data result from a continuous, changing process and this process takes place under specific conditions existing in a definite time period.[53] If past data are used, inefficiencies of prior periods will be reflected in the regression line.[54] In addition extended use of historical data may lead to distorted results due to intervening changes in conditions.[55] The cost structure along with the related cost "depend essentially upon the starting-point of the production changes as well as upon the amount of the volume variation during a specific period of time. Furthermore, the direction of the output variations will have a strong influence upon the slope of the cost curve."[56]

A fourth possible dilemma arising from the process of fitting a trend line should be mentioned — the subjective element which may be interjected by the unsophisticated statistician in making his choice of the formula to be used, i.e., is the relationship shown in the data to be handled in terms of one of the linear regression models or is it to be

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[52] Ibid.

[53] Ibid., p. 22.

[54] Gordon Shillinglaw, Cost Accounting Analysis and Control (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1967), pp. 11-12.

[55] Separating and Using Costs as Fixed and Variable, pp. 11-12.

[56] C. Weber, pp. 22-23.

analyzed by means of a curvilinear model? In making this choice of technique, he may operate under his preconceived, although logically determined, notion as to what he believes the trend will look like.[57] Thus, the objectivity of the results of the regression analysis lies mainly in the use of mathematics to fit the trend line, but the problem of subjectivity may still exist in the choice of the appropriate formula and, therefore, affect the results. This problem tends to arise when the user of regression analysis is not aware of, or is uncertain as to the use of, the various tests which may be employed to find the function which best fits the actual relationship shown by the data.

A final problem in connection with regression analysis procedures, which may be overcome easily, relates to the calculations themselves. They can be very laborious and time-consuming unless a computer is available. The process may also be expensive because the underlying data are often subject to considerable modification, in order to meet the fundamental <u>ceteris paribus</u> conditions.[58] Such modifications can range from the complete elimination of atypical data to manipulation of the data; both types of corrections may introduce subjectivity into the results.[59]

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[57] Bhada, Some Implications . . . , p. 136.

[58] C. Weber, pp. 7-8.

[59] Ibid., p. 22.

Correlation Analysis

The results obtained under the visual curve fitting or the regression procedures must meet two conditions if they are to be considered reasonably accurate estimates: "(1) all the plotted points [should] appear on the regression line, and (2) the conditions which operated in the past from which the data have been taken, [should] operate in the future."[60] These conditions are rarely, if ever, exactly met in practice, and a technique is needed to measure the effects of failure to achieve them on the cost analysis.

In statistical analysis a criterion does exist which can be used to test the goodness of fit of the regression line to the data, and this helps temper one problem mentioned earlier -- the ability of regression analysis to fit a line to any set of data. This criterion is the correlation coefficient which "measures the extent to which the output variable explains changes in cost."[61] This figure may be calculated as a by-product of the regression analysis, since the same data are used for both sets of equations. The results obtained by carrying out this additional analysis need to be interpreted carefully. Even if a fairly high correlation coefficient exists in a particular situation, the existence of a "cause-and-effect" relationship should not be assumed.[62]

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[60]Batty, p. 228. The implications of the failure to meet the latter of these two conditions was discussed on page 62, above.

[61]Dopuch and Birnberg, p. 55. [62]C. Weber, p. 8.

Correlation analysis may also be useful in the problems arising in the selection of the proper volume base when used in connection with multiple regression analysis. By means of the multiple regression analysis, the effect of several cost-causing factors may be considered, and correlation analysis may then be used to determine the ones most significantly related to cost. Correlation analysis may be employed also to indicate how much reliance may be placed on the actual separation of the costs which is calculated using the selected volume base.[63] This is important in the setting of overhead standards because variable overhead costs are viewed generally as being related to a base expressive of physical activity, such as direct labor hours or machine hours. If there are several bases which bear a relationship to a particular item of overhead cost, correlation analysis may help in determining which one should be used.

It was mentioned at the beginning of this chapter that, in order to set up the product overhead standard for each category of costs, all overhead costs will need to be classified as being either fixed or variable. The use of statistical techniques, e.g., regression analysis, represents an attempt to make the resulting classification as objective as possible while correlation analysis tries to measure the reliability of the results. Within certain limitations, these purposes are attained, but statistical techniques, by dealing with the past, bring back a situa-

[63]Gynther, p. 32.

that standard costing was intended to alleviate Because of this reli-
ance on the past, statistical analysis should be viewed as only the first
step in any analysis

> Mere satisfaction of a mathematical formula does not guarantee that the flexible budget allowance will be reasonable. Budget allowances should be based on the best estimate of future relationships and these may or may not coincide with the relationships indicated by mathematical equations derived from historical data [64]

Impact of Statistical Analysis Upon Standard Costs

Statistical techniques employed to separate mixed and variable costs are an improvement over the accounting method in that they may help to create the establishment of a more precise rate of variability of the cost, through the slope of the regression line, and the amount of the fixed cost through the constant term They may also increase the likelihood that cost designations will be changed from one period to the next as the cost item itself changes from fixed to variable or semi-variable for example Correlation analysis may help in the determination of the most appropriate activity base to which a particular variable overhead cost will be tied This would be especially useful where there are several alternative bases under consideration.

Mixed costs generally are overhead costs, the components of which will be handled differently for various purposes depending on whether they are fixed or variable This is particularly true when a standard

[64] Shillinglaw (rev ed), p 393

cost system is in use. The main concern of the present chapter is the construction of standard overhead rates where usually there is one rate for the variable costs and a separate one for fixed costs. Ordinarily standard variable overhead costs are attached to the product on the basis of a constant dollar amount per some chosen volume base, e.g. direct labor hours.[65] Fixed overhead costs are applied on a similar basis, but their rate per unit will be based upon the particular capacity utilization which is budgeted, or normal, for the period under consideration.[66] These rates are then used in variance analysis, as discussed in Chapter IV as well as for product costing and budgeting. There are, however, a number of other areas utilizing standard costs which require a separation of the mixed costs into their components. These include flexible budgeting, direct standard costing and linear programming (as discussed in Chapter V, pp. 128-139.)

The word "precise" has come up several times in the discussion of the results of regression analysis. The increased precision achieved in the separation comes about, initially at least, through the use of a mathematical formula rather than judgment or past experience. Additional precision may be achieved by developing various other statistics and analysing the results in the light of the new information.[67] The

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[65]Horngren, pp. 272-273.   [66]*Ibid.*, p. 276.

[67]Some of these additional statistics which might be calculated are the correlation coefficient, the standard error of the estimate, t ratios, and coefficients of partial correlation (where multiple regression is

employment of most of these tests will depend upon the analytical sophistication of the user.

The main impact upon the decomposition of mixed costs into their two components has, thus far, come from the use of least-squares analysis which provides a clear dichotomy between fixed and variable. A lesser influence has been developed from multiple regression. This latter area, however, has a potential effect in that it may help in the establishment of causes for variances in these costs, since a number of independent variables are considered. It may also enable the analyst to predict the effect upon potential costs if there is a change in one of the independent variables so that a more forward looking approach may be applied to the establishment of standards. In any event, whether or not it is directly employed in standard setting, a knowledge of multiple regression analysis heightens the understanding of the accountant and the analyst with respect to problems of cost variation.

Summary

This chapter looked into the techniques used in separating mixed overhead costs into their fixed and variable components. After reviewing two of the more traditional techniques for carrying out the decomposition process, statistical techniques involving scatter-graphs and/or regression analysis were discussed along with their limitations.

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being used)

The use of correlation analysis as a test of the reliability of the regression analysis was brought in as well as its use as an aid to finding the appropriate independent variable to which the dependent variable should be related.

A question was posed in Chapter II as to whether the use of statistical techniques in setting standards would help standard cost systems overcome the defects which were felt to exist in the historical cost systems.[68] The statistical procedures of the present chapter, although relying on historical data, provide a mathematically precise and objective technique for separating the mixed overhead costs into their fixed and variable components which may also lead to more frequent updating of the standards. Thus, there is improvement if such techniques are utilized and their limitations clearly understood.

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[68] See page 19

IV VARIANCE ANALYSIS, CONTROL, AND STATISTICAL CONTROL MODELS

"Variances constitute a connecting link between standard costs and actual costs."[1] They are a prime element of the control function of standard costing and are generally calculated after specific time periods have elapsed, e.g., a month. The most important type of cost control which should exist in any system is that exercised before the fact ... "preventive cost control." Implementation of such a process necessitates the use of standards which are kept current.[2] A procedure for this was discussed in Chapter II -- learning curves.[3]

There are several things management should know in addition to the size and type of variance before it can exercise improved control over costs ... "where the variances originated, who is responsible for them, and what caused them to arise."[4] Thus, the significance of variances must be determined in the light of these factors.[5]

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[1]"The Analysis of Manufacturing Cost Variances," in Thomas, Jr., p. 593.

[2]Ibid., p. 594. [3]Pages 26-46.

[4]"The Analysis of Manufacturing ...," p. 595. [5]Ibid.

This chapter will be concerned with the various statistical cost control techniques which have been suggested as ways to improve standard cost variance analysis, particularly with reference to the determination of sources, causes and, perhaps, responsibility. Both a brief review of traditional variance analysis procedures and the general topic of the meaning of statistical cost control will be presented as background for an examination of the impact of such techniques as control charts, regression analysis, modern decision theory including Bayesian statistics, and controlled cost, upon standard costs.

Traditional Variance Analysis

An essential feature of variance analysis is the availability of some base capable of being used for comparison.[6] Under the forms of cost accounting existing prior to the acceptance of standard costing only one 'interesting' cost variance could be calculated -- the variation in actual costs between periods. These costs generally could not be used to determine the degree of efficiency existing during the periods being compared and, thus, the variations can be used only to indicate the direction of the trend of the operational performance, not to act as an index of efficiency.[7]

Standard costing, by recording costs on a "dual base," i.e., both the actual and the standard cost are recorded, helps to provide more

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[6]Harrison, p. 228.

[7]*Ibid.*

meaningful variances.[8] No longer is the analysis limited to interperiod comparisons, but the actual cost incurred during a period can be contrasted with the standard established for that cost. The discovery of variances between standard and actual costs is an important way of disclosing intraperiod operating inefficiencies and also acts as a form of "management by exception" in that only variances are reported to management.[9]

Cost control may be considered a basic management tool.[10] The N.A.A. defines the objectives of cost control as follows: "cost control has as its objective production of the required quality at the lowest possible cost attainable under existing conditions."[11] The idea of using standard costs to achieve this objective has existed for some time. Harrison based his original standard cost system upon five principles, at least three of which bring out the concept of control.[12]

1. Determination of proper cost before the articles, goods or services are produced.
2. Recognition of the fact that variations from standard costs will inevitably arise in practice. [The variation of the cost of the

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[8] Ibid.

[9] Ibid., pp. 228-229.

[10] Feng-shyang Luh, Controlled Cost: An Operational Concept and Statistical Approach to Standard Costing (Ph.D. Dissertation, Ohio State University, 1965), p. 1.

[11] "A Re-Examination of Standard Costs," in Solomons, Studies in Costing, p. 443.

[12] L. P. Alford, "Cost Control," Mechanical Engineering, LVI (1934), p. 467, as quoted by Upchurch, p. 27.

> same article at different times constitutes the important point, not only in the proper understanding but in the appreciation of costs. The ability to master this point and to figure estimates or predictions of costs from a standard under varying conditions gauges the comprehension of the meaning and value of practical value.[13]

3. Analytical procedures to be applied to these variations to determine their causes.
4. Application of the management law of exceptions. [Management efficiency is greatly increased by concentrating managerial attention solely upon those executive matters which are variations from routine, plan or standard.][14]
5. Application of the management law of operating rates. [Operating performance is controlled most directly through control of the rates of expenditure for labor, materials, and expenses.][15]

Two methods of variance analysis were, and are, used: one signals the need for investigation when the dollar amount of the variance exceeds a predetermined cut-off point; the other looks at cost ratios.[16]

An early writer, Cecil Gillespie, presented a discussion of the types of variances which may be calculated. These tend to employ the first type of investigation decision, above. His system, which is based upon a fixed budget, closely resembles the conventional procedures found in many managerial accounting textbooks today. A numerical comparison of Gillespie's procedure with the more recent analysis

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[13] _Ibid._, pp. 27-28.

[14] L. P. Alford, _Laws of Management_ (New York: Ronald Press, 1941), p. 115, as quoted by Upchurch, p. 28.

[15] _Ibid._, p. 29.

[16] Robert Wallace Koehler, _An Evaluation of Conventional and Statistical Methods of Accounting Variance Control_ (Ph.D. Dissertation, Michigan State University, 1967), p. 15.

techniques is presented in Appendix B

Net variation from standard costing may be analyzed into these price and quantity factors:

(a) Price variation, consisting of
   (1) Net variation between actual and standard price of materials used
   (2) Net variation between actual and standard price of labor used
   (3) Net variation between actual and budget factory expense for month
(b) Quantity variation, consisting of
   (4) Net variation between actual and standard quantity of materials for the month's production, priced at standard
   (5) Net variation between actual and standard quantity for labor for the month's production, priced at standard
   (6) Net variation between budget hours of factory expense for month and actual for the month's production priced at standard
   (7) Net variation between actual hours of factory expense and standard hours for the month's production, priced at standard [17]

An early exponent of cost ratios was Camman. These ratios had a number of uses, including "the measure of performance," "index characters for comparison with others in terms of common denomination," and "barometric symbols indicating the rate and direction of the trends."[18] Actual ratios are compared with expected ratios and in this way not only show how closely the expected results were realized, but also provide a means for calculating any realized gains or losses.[19] This technique is more practical than the predetermined cut-off point procedure because it employs a relative, rather than an absolute, con-

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[17]Gillespie (1935), p. 34.

[18]Camman, p. 93.

[19]*Ibid.*, pp. 93-94.

cept, thus where large amounts of cost are involved, the absolute variance, price or quantity may be greater before warranting an investigation A predetermined cut-off point would not permit such flexibility [20]

The traditional accounting control model, which has been the one typically presented in managerial accounting textbooks, may be summarized as follows the standard cost is developed as a point estimate from which deviations are calculated control is based on a subjective decision regarding the determination of the cut-off point and it is carried on after the fact [21] The subjectivity does not lead to a clear differentiation between the causes of the variation i e , are they caused by factors under management control or by something beyond anyone's control?

Three Problems of Traditional Techniques

A main concern of the accountant in the traditional variance analysis procedure is to determine first if the deviation is favorable or unfavorable - a mathematical procedure Then he must decide, based on some subjective rules whether or not to investigate [22] The first problem is in the dependency on subjectivity The techniques which

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[20]Koehler, p 16

[21]Mohamed Onsi 'Quantitative Models for Accounting Control,' The Accounting Review XXXXII (April 1967), p 322

[22]Louis A Tuzi Statistical and Economic Analysis of Cost Vari

follow aim to remove the subjective element from the decision process or supplement it with a more scientific rule

The second problem which statistical technique may help to overcome is that of compensating variances An example of how such a situation might occur is the case of a department which handles several operations One operation might incur a significant (controllable) variance during the period which is off-set by the variances due to chance (noncontrollable) causes in the other operations, assuming variances are aggregated and reported for the department as a whole If the variance is determined by operations a similar problem may develop because of the time period over which the data are accumulated It is necessary to try to eliminate these "off-set" or "average-out" problems in order to expedite the detection of the assignable causes of deviation [23]

The third and final problem to be considered is found in the investigate/do-not-investigate decision The conventional analysis procedures by using an arbitrary cut-off point in making this decision, run the risk of failing to investigate when it is warranted Type I error or investigating when it is not required Type II error [24]

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ances (Ph D Dissertation, Case Institute of Technology, 1964), p 49

[23] Koehler, p 23

[24] These errors are generally defined in terms of the acceptance or rejection of a "null" hypothesis. In this situation, the null hypothesis might be stated variance X should be investigated Thus a Type I error implies that the null hypothesis has been rejected when it is true, a Type II error then is the acceptance of the null hypothesis when it

Statistical Cost Control

Control System Requirements

The main purpose of cost control is the maximization of operational efficiency. This is done by looking for any abnormalities in performance which would indicate that the process is out of control due to assignable causes.[25] There are at least three objectives which should be met by any cost control system if it is to be effective:

1. Current operating efficiency should be maintained and deviations from it should be identified.
2. Any indication of an impending crisis should be disclosed.
3. The existence of any means by which current operating efficiency may be improved should be revealed.[26]

Only the first objective is met by traditional standard cost variance analysis which assumes that the standard for a particular operation is stable and, therefore, when abnormalities arise requiring the attention of management, this implies that there has been a 'significant' deviation from the standard.[27] The first objective will also be met by the various statistical control procedures in that these techniques will sig-

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as false. Schlaifer, p. 608.

[25] Luh, p. 37.

[26] F. S. Luh, "Controlled Cost: An Operational Concept and Statistical Approach to Standard Costing," The Accounting Review, XXXIII (January, 1968), p. 123.

[27] Ibid.

nal deviations from some expected, or mean, value. That they might also fail to meet the other two objectives will be demonstrated in the following sections.

In addition to the three objectives, there are some "practical requirements" which any chosen control process should meet:

1. The presence of assignable causes of variation should be indicated.
2. The means by which such causes are indicated should also provide a process by which the causes can be discovered.
3. The criterion should be simple, but also "adaptable in a continuing and self-correcting operation of control."[28]
4. The possibility that assignable causes will be looked for when, in fact, none exist should not exceed some predetermined value.[29]

Meaning of Statistical Cost Control

The verification of a system considered to be under statistical con-

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[28] Walter A. Shewhart, Statistical Method from the Viewpoint of Quality Control (Washington: The Graduate School, The Department of Agriculture, 1939), p. 30.

[29] Ibid. One might also consider characteristics which the operation to which statistical cost analysis is to be applied, should possess:

1. "an operation must be repeated a number of times",
2. "an operation should be independent of other operations as far as possible",
3. "an operation should be a functional unit",
4. "an operation should have only a few major factors which affect its cost." L. Wheaton Smith, Jr., "Introduction to Statistical Cost Control," N.A.C.A. Bulletin XXXIV (December, 1952), pp. 512-513.

trol is that any variations which may occur are attributable only to chance factors.[30] A chance factor may be defined as "any unknown cause of a phenomenon."[31] This determination is made primarily by means of the creation of control limits which would define the range of deviations felt to be caused by random factors.[32] If a variance were to fall outside the control limits, it would signify that the system is out of control and the cause of the deviation should be investigated.[33] When an operation is considered to be under statistical control, which it must be prior to the application of the statistical procedures to be discussed below, it is felt to be a stabilized operation with cost variations falling within the limits most of the time and the probabilities of this occurring can be approximated.[34]

There are two circumstances which can lead to a system being out of statistical control: (1) "There may be no constant 'cause' system for the operation," meaning that there is variation beyond the limits considered to be normal in some factor or factors,[35] and (2) there is a failure to include all of the important factors or their interactions in

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[30]Crowningshield, p. 797.

[31]W. A. Shewhart, Economic Control of Quality of Manufactured Product (New York: D. van Nostrand Company, Inc., 1931), p. 7.

[32]Crowningshield, p. 797. [33]Ibid.

[34]Smith, p. 515.

[35]Ibid. A "constant cause" system is one in which "the factors affecting the results of the operation probably have not changed or varied outside their usual ranges." Ibid., p. 511.

the analysis.[36]

The Normality Assumption

It is generally assumed that the probability distribution from which the samples are drawn is a normal one. Although this is a practical assumption, it may not be a valid one. However, as long as there is no significant deviation from the shape of the normal distribution, the results will still be useful, although less precise, than if the true distribution were used.[37]

The typical shape of the normal curve shows a concentration of frequencies of observations about a single central point with small numbers of observations at the extremes -- a monomodal, bell-shaped curve. There are some distributions which closely resemble this pattern in that there is a concentration of points about a mean, but the frequencies at the extremes are not distributed symmetrically. This type of distribution is called skewed.[38] There is a feeling that many accounting costs tend to have a skewed, rather than normal, distribution.[39]

The problems involved in the estimation of an unknown, possible non-normal distribution may be overcome mainly by using the distri-

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[36] Ibid., p. 515.

[37] Frank R. Probst, The Utilization of Probabilistic Controls in a Standard Cost System (Ph.D. Dissertation, University of Florida, 1969), p. 25.

[38] Turn, pp. 34-35. [39] Ibid., p. 19.

bution of sample means rather than the distribution of the individual observations. The former tends to approximate the normal distribution, even if the latter have a non-normal distribution, if two theorems are applied: the Law of Large Numbers and the Central Limit Theorem.[40]

Accounting Implications

The characteristics of the traditional accounting control model were presented on page 75. In contrast, were this model based upon the concepts of classical statistics, it would have the following properties:

(1) Standard cost is equal to the mean of a normal probability distribution.
(2) Standards are developed as ranges, not point estimates.
(3) The allowable deviation is represented by the size of the control limits; and
(4) Investigation is exercised when one or more consecutive observations lie outside the central limits.[41]

Two types of deviations from standard are assumed to exist under such

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[40]Ibid., p. 35. These theorems can be found in: William Feller, An Introduction to Probability Theory and Its Applications, Vol. 1 (2nd ed.; New York: John Wiley & Sons, Inc., 1957), pp. 228-229.

"Law of Large Numbers. Let $\{X_k\}$ be a sequence of mutually independent random variables with a common distribution. If the expectation $\mu = E(X_k)$ exists, then for every $\epsilon > 0$ as $n \to \infty$

$$P\left\{\left|\frac{X_1 + \cdots + X_n}{n} - \mu\right| > \epsilon\right\} \to 0 \cdots$$

Central Limit Theorem. Let $\{X_k\}$ be a sequence of mutually independent random variables with a common distribution. Suppose $\mu = E(X_k)$ and $\sigma^2 = Var(X_k)$ exist and let $S_n = X_1 + \cdots + X_n$. Then for every fixed

$$P\left\{\frac{S_n - n\mu}{\sigma n^{1/2}} < \beta\right\} \to \Phi(\beta)$$

where $\Phi(x)$ is the normal distribution. . . ."

[41]Onsi, pp. 321-322.

a model "chance" variances from random factors and assignable deviations due to "systematic" causes. Only the latter type should be investigated.[42]

The traditional concept of standard costs, with its single point estimate, assumes that there is no distribution of cost around the standard. Thus, every variance should be explained. There also is no systematic procedure included for revising the standards based on the empirical evidence. These difficulties are reduced by the introduction of "probabilistic standards."[43] To do this, the managerial accountant has to develop systems based on expected costs, not the traditional actual cost basis.[44] The assumption of a normal distribution of the deviations from the expected cost, or mean, leads to the further assumption that the unfavorable and favorable variances will be distributed equally, and without pattern, around the standard as long as they are due to random causes.[45]

This classical statistical accounting control model and its implications will be discussed more fully in the following section on control charts.

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[42]*Ibid.*, p. 322.

[43]Zenon S. Zannetos, "Standard Costs as a First Step to Probabilistic Control," *The Accounting Review*, XXXIX (April, 1964), pp. 297-298.

[44]*Ibid.*, p. 296.

[45]Onsi, p. 322.

Control Charts

The concept of statistical control leads to the use of a range of costs rather than a single value for purposes of comparison and control limits to designate the band of costs felt to be acceptable chance variations from the mean.[46] Any costs which exceed either limit are deemed to have been caused by nonrandom factors, therefore controllable, and should be investigated.[47] A basic assumption for such procedures is that the costs being analyzed are "generated by a well-behaved underlying process."[48]

Chebyshev's Inequality[49]

This is a generalized control limit type procedure which may be used for the purpose of accounting control when the distribution of the costs is unknown. Basically the procedure permits the analyst to determine how significant, in probability terms, a variance from standard is "by finding the lower bound (or upper bound) of the probability that a variance will be less (or greater) than a certain number of standard deviations."[50] The analyst will be able to ascertain what percentage of

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[46] Luh, The Accounting Review, p. 123. [47] Ibid., pp. 123-124.

[48] Hasgrave, p. 856.

[49] "Theorem: Let X be a random variable with mean $\mu = E(X)$ and variance $\sigma^2 = Var(X)$. Then for any $t > 0$

$$P\{|X - \mu| \geq t\} \leq \sigma^2/t^2$$ " Feller, p. 219.

[50] Zannetos, p. 297.

the variances which occur should be expected, assuming the process is in control, and which require action.[51]

This technique is more of a theoretical tool than a practical one. "The importance is due to its universality, but no statement of great generality can be expected to yield sharp results in individual cases."[52] Chebyshev's inequality uses individual observations of cost, the distribution of which may be unknown. This accounts for its universality of application.

As long as the cost variations have the same, although perhaps unknown, distribution and a finite variance can be computed, then the Central Limit Theorem in combination with the Law of Large Numbers may be applied to develop an almost normal distribution from the sample means.[53] When this is possible, more practical techniques of cost control may be used. However, Chebyshev's inequality may be employed to obtain a rough approximation of the appropriate probability law as long as the mean and variance of the random variable are obtainable and with standards this latter condition may be ignored since the parameters may be developed empirically. Such approximations often are adequate for the analysis of accounting data.[54]

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[51] Ibid.

[52] Feller, p. 219.

[53] Schlaifer, p. 426.

[54] Zannetos, p. 297.

Quality Control Chart Concepts

The concepts of quality control charts were set forth in the 1930's by W. A. Shewhart. He felt that there were two main characteristics of control, "variability" and "stability": variability because the quality being analyzed must vary in order to require control; stability because the variations should occur only within predetermined limits. Shewhart defined the problem as follows: "How much may the quality of a product vary and yet be controlled?"[55] This problem is equally applicable to situations where cost, rather than quality, is being controlled.

A basic assumption in the establishment of a statistical control limit is that the standard cost is equal to the average cost as determined from a number of observations felt to be representative of "behavior under standard conditions."[56] Once this mean is determined the control limits can be established by means of a formula and a set of tables. An additional assumption is that the distribution of the data is normal. To ensure this, the sample means are plotted rather than the single observations.[57]

Two types of control charts may be established. The one most typically used is the $\overline{X}$ chart which plots the sample means. The other,

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[55]Shewhart, Economic Control . . ., p. 6.

[56]Shillinglaw (rev. ed.), p. 353.

[57]The limits are calculated as $\overline{X} \pm 3\sigma$, but if $\overline{X}$, R and the sample

the R chart plots sample ranges. This latter chart, which rarely goes out of control and thus may be ignored in future discussions, is used to control variability within the process. However, process variability also may be controlled with the $\overline{X}$ chart when it is subjected to periodic revision.[58]

The control charts are initially established from past data on cost variances and will be useful in determining if the process was in control. Once the assignable causes of variation, or out-of-control points, have been erased from the data, the control limits should be revised. These new boundaries may be used to analyze future data only if the process is in control and remains so. Periodic revisions, however, are necessary to reflect any permanent changes made in the firm's operating policy.[59] Shewhart defined the desired conditions of control as follows: ". . . maximum control . . . will be defined as the condition reached when the chance fluctuations in a phenomenon are produced by a constant-cause system of a large number of chance causes in which no cause produces a predominating effect."[60]

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size are known, a table called "Factors for Determining from $\overline{R}$ the 3-Sigma Control Limits for $\overline{X}$ and R Charts" may be used to determine the limits using the following formula for the $\overline{X}$ chart: $\overline{X} \pm A_2\overline{R}$

[58]Probst, The Utilization of . . ., p. 56.

[59]Tuzi, pp. 79-80.

[60]Shewhart, Economic Control . . ., p. 151.

Several signals indicating the need for a possible investigation may be obtained from the use of a control chart. The first, and most obvious, is the existence of samples which fall outside the limits, thus probably indicating that some nonrandom, therefore controllable, factors are affecting the process.[61] It is also possible that there may be a run of points on one side of the center line.[62] If such a run is determined to be statistically significant, it may be an indication of a shift in the process average due to a "force acting on the data outside the constant-cause system."[63] Third, a bunching up of points near a control limit, or some secondary limit, e.g., the 2σ limit, might occur. Or, finally, a trend may be seen in the points.[64] These latter warnings would also signal a change in the process average due to nonrandom factors.

The approach of quality control charts for cost control is generally felt to be applicable only to labor costs but it may be used also for material costs, since samples of these latter costs are obtainable on a daily, or shorter, basis. If the time horizon is expanded to a monthly basis for the purpose of sampling, the procedure may also be employed

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[61]Teal, p. 146.

[62]A run is "any consecutive sequence of points falling above or below the process average." Koehler, p. 61.

[63]Teal, p. 146.

[64]Luh (Ph.D. Dissertation), p. 22.

in the analysis of overhead items [65]

Regression Control Charts

One of the earliest articles suggesting the use of regression analysis as a technique for variance analysis was written by Joel Dean in 1937 in which he suggested multiple regression analysis of past variances as a way of segregating the uncontrollable deviations from the controllable [66] Since that time there have been a number of articles which present the results of regression analysis, simple linear or multiple, as applied to a specific cost control situation [67]

In order to use a statistical technique such as regression analysis a relationship must be shown to exist between the variance (dependent variable) and some unknown factor(s) (independent variable(s)) [68] The scatter-graph as described in Chapter III, may be employed for this

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[65] Probst The Utilization of p 32

[66] J Dean, "Correlation Analysis of Cost Variation " The Accounting Review, XII (January, 1937), p 55

[67] For example A W Patrick, "A Proposal for Determining the Significance of Variations from Standard " The Accounting Review XXVIII (October, 1953) pp 587 592 Eugene E Comiskey "Cost Control by Regression Analysis," The Accounting Review, XXXXI (April, 1966) pp 235-238; Robert A Knapp "Forecasting and Measuring with Correlation Analysis" In Contemporary Issues in Cost Accounting Eds Hector R Anton and Peter A Firmin (2nd ed ; Boston Houghton Mifflin Company 1972), pp 107-120 Edwin Mansfield and Harold H Wein "A Regression Control Chart for Costs," in Studies in Cost Analysis Ed David Solomons (2nd ed ; Homewood, Ill ; Richard D Irwin, Inc 1968) pp 452-462

[68] Patrick, p 588

purpose when the model has only two variables One possible relationship which has been suggested for use is that between consumption and variation [69] Also, a regression line may be fitted to the scatter of points The degree of scatter around the trend line, for purposes of variance analysis, may be measured by means of the standard error of the estimate which is a 'measure of the statistical variation which has not been explained by the estimating equation '[70]

It is still possible to establish "control limits" around the regression line These limits, although calculated differently, will serve the same purpose as the control limits determined for the more typical quality control chart [71] The standard error of the estimate is used for this purpose.[72] As in the quality control chart techniques, the observations about the regression line should be scattered randomly and points falling outside the "control limits" or showing a possible trend act as signals of a change in the variation pattern [73]

Generally the data plotted on a regression control chart are not sample means, but individual observations Therefore the distribution should be more nearly normal than for the quality control chart A second difference is in the 'measure of central tendency." In the quality control chart, the mean, which is developed from the parameters of the system, is used in the regression version, a line or plane created

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[69] Ibid , p 589 [70] Ibid. [71] Ibid

[72] Ibid , p 591 [73] Ibid.

from estimates that are subject to error is employed.[74]

A further difference between the two types of control charts -- quality and regression -- is the lack of a time chart when the regression control chart is used.[75] Visual presentation, which is easier to achieve with the quality control chart, makes the process more understandable to those using it, and makes the warning signals readily apparent.[76] By plotting the sample means and looking for trends or runs, the analyst is informed of the possible need for a revision due to a change in the process average.

There are three characteristics of multiple regression analysis which make it a useful tool for cost control:

1. Individual (e. g., monthly) errors are minimized and off set one another to maximum extent, leading to a minimum total period (e. g., year) error.
2. Statistical by-products provide the capacity to predict limits of acceptable error, or variance, both monthly and year to date and thus signal the need for second looks.
3. Through the predicting equation, causes for forecast error, or budget variance, can be quantitatively identified.[77]

If multiple regression is used, it is possible, by a trial and error process, to test various combinations of operating costs and factors felt to affect them in order to find the proper combination of independent variables which explains most of the cost variation.[78]

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[74] Mansfield and Wein, p. 461.

[75] Ibid.

[76] Koehler, p. 61.

[77] Knapp, p. 108.

[78] Robert E. Jensen, "A Multiple Regression Model for Cost Control -- Assumptions and Limitations," The Accounting Review XXXXII (April, 1967), pp. 267-268.

A procedure such as the regression control chart is open to several objections as well as possessing advantages. Among the advantages are the ability to isolate the explainable parts of the variance which would help in determining responsibility for the controllable variance, and the possibility of eliminating some of the off-set or average-out problems.[79]

Despite these advantages there are some serious objections. One has been mentioned before in connection with regression analysis -- the technique is based on the past: the regression line and its coefficients are determined from past variances and relationships. Second, the segregation of the variances into controllable and noncontrollable types is not complete since it is limited by the amount of the relationships which can be measured statistically. Finally, the variances are only measured by the procedure, not controlled.[80]

A major fault in the regression control chart, which also exists in the conventional quality control chart, is the fact that only a signal is provided that something is unusually wrong with a particular observation or sample mean. No data are provided relating to the cause of the excessive variance or how to improve performance.[81] Thus, only the first objective of a control system is met by these procedures, the same as in the conventional standard cost variance analysis techniques. An additional failure of both control chart techniques is the lack of consid-

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[79]Dean, p. 60. [80]Ibid., p. 59.

[81]Mansfield and Wein, p. 462.

eration of the cost of investigating the variance

Illustration of regression control limits

Let $X = \{x_i\}$ be a set of observations of the independent variable and $Y = \{y_i\}$ represent the set of dependent variables associated with these observations. As long as the problem involves only two variables it is possible, if desired, to draw a scatter diagram of the points, $x_i y_i$. This chart may be helpful in determining the form of the equation to be used in fitting the regression line and the control limits.

If it is assumed that the desired model is linear, the trend line equation may be expressed as $Y = a + bX$ where the coefficient b represents the slope of the line and the constant, a, establishes the intercept of the line with the vertical axis (see Figure 4).

Once these coefficients are developed, they may be used to calculate the standard error of the estimate (also referred to as standard deviation from regression, conditional standard deviation). The formula for this may be written in several ways, such as

$$S_{y\cdot x} = \sqrt{\frac{\sum(Y - Y_e)^2}{n - 2}} \qquad \text{where } Y_e = a + bX$$

The result for $S_{y\cdot x}$ may be used much like the standard deviation in establishing control limits around the regression line. Thus, $\pm 1S_{y\cdot x}$ includes 68 per cent, $\pm 2S_{y\cdot x}$ 95 per cent, and $\pm 3S_{y\cdot x}$ 99.7 per cent of all observations.[82]

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[82]Herbert Arkin and Raymond R. Colton, Statistical Methods (4th ed. revised; New York: Barnes & Nobles, Inc., 1956), pp. 77-78.

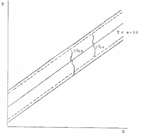

Figure 4 Example of a Regression Control Chart

When using a model of this type for control purposes, variations due to random causes should fall within the $\pm 3S_{y \cdot x}$ limits most of the time and any variations falling outside these limits are probably not due to random causes and, thus, require investigation

Impact on Standard Costs

Quality control chart techniques are quite closely related to the more traditional standard cost variance analysis techniques The major impact of these procedures on standard costing is the development of a band of costs to replace the conventional single point estimate [83] Thus, because of the control limits, helps bring to light variances which may be caused by nonrandom factors. These are the deviations which should be brought to the attention of management The costs which are used in the regression control charts are estimates which "are not equivalent to carefully constructed standard costs," and, thus, do not have an important impact [84]

Because of the frequency of the observations needed in developing the sample means, weaknesses in the cost structure are brought to management's attention sooner -- not at the end of the month, for example, but daily or weekly The more timely analysis helps prevent the overlooking of some classifications [85] Usually there are a number of off-

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[83]Li, p 629.

[84]Cemiskey, p 238

[85]Edwin W Gaynor "Use of Control Charts in Cost Control " in Thomas Jr , p 836.

setting plus and minus variations which will be overlooked when the analysis is made after a long time interval. The more frequent observations of the control chart ensure that many of these deviations will be noted.[86] The detection of the cause of the variance also will be easier.[87] The only information which may not be directly ascertainable from the control charts is the establishment of who is responsible for the variance and the cost involved in making the investigation.

The time series plot of the sample means developed with quality control charts informs the analyst of the possible need for a revision due to a change in the process average by allowing him to look for trends or runs in the series. Thus, a new signal is given as to when the standard needs to be revised, not just the passage of a sufficient length of time of the occurrence of a substantial irregularity. Also, quality control charts, once they have become operational, may be subjected to regular revisions which may, through sample observations, not only eliminate assignable causes, but may also show the effects of the learning curve.[88]

The use of either type of control chart will impose one requirement on the cost accountant. Both the standard and actual costs involved in the variance analysis must be "free of contamination."[89] The cost data

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[86]Horngren, p. 856.   [87]Ibid., p. 857.

[88]Probst, The Utilization of ..., p. 32.

[89]Tuzi, p. 28.

cannot be aggregated; "to achieve pure basic data, the data must represent a single activity for a single production effort."[90] Therefore, although the accountant may need to aggregate the product costs for inventory purposes, for example, he must be able to provide the individual elements for variance analysis.

Other Statistical Control Models

Two general types of models will be considered in this section, both of which are no longer part of the classical statistical model: modern decision theory with an emphasis on the Bayesian decision rule and Luh's controlled cost model. The Bayesian model has two advantages over the classical statistical method of analysis:

1. all possible alternative parameters may be considered rather than just one.
2. the causes of the variances may be identified more easily after the posterior probabilities are determined.[91]

Modern Decision Theory Models

Earlier in the chapter the characteristics of the traditional and classical statistical accounting control models were described.[92] This section is concerned with a further refinement in which some of the ideas of modern decision theory are used, especially Bayesian analysis

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[90] Ibid.

[91] Koehler, p. 133.

[92] See pages 75 and 81.

The point estimate standard cost is replaced by an expected value concept; control is carried out through a combination of scientific analysis and personal judgment and it is exercised before the fact.[93] "The decision to investigate will be a function of the probability that the operating segment is operating out of control, the costs of operating out of control and the costs of investigation."[94] Figure 5 presents a comparison of the three types of control models, taking into account not only the nature of control but also the criteria of control and the requirements needed before exercising control.

A number of writers have proposed control models which may be considered under the heading of modern decision theory. One of these models was proposed by Bierman, Fouraker and Jaedicke. This model is based primarily on a desire to minimize the cost of investigation. The authors add a new measure to the investigate/do-not-investigate decision: "the probability of the variance occurring from random causes," which is then used to arrive at a cost for each of the two possible actions.[95] The standard cost for a particular item will be the expected value of the actual cost, and the determination of this expected

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[93]Onsi, p. 322.

[94]Robert S. Kaplan, "Optimal Strategies with Imperfect Information," The Journal of Accounting Research, VII (Spring, 1969), p. 32.

[95]Harold Bierman, Jr., Topics in Cost Accounting and Decisions (New York: McGraw-Hill Book Company, 1963), p. 15.

Elements	The Traditional Accounting Control Model	Accounting Control Model Based on Classical Statistics	Accounting Control Model Based on Decision Theory
Nature of Control	1 Based on a point estimate 2 Based on management judgment 3 Developed after all facts are known 4 Developed as a deterministic feedback control model	1 Based on a range estimate 2 Based on scientific analysis 3 Developed as a preventive control model 4 Based on stochastic feedback control processes	1 Based on the expected value of [illegible] that will be obtained based on investigation 2 Based on judgment and scientific analysis 3 Developed as a preventive and time [illegible] control model based on the sample [illegible] 4 Based on stochastic and adaptive feed[illegible] control processes
On basis of Control (Investigate or do not investigate	1 If the absolute size of a deviation is large 2 If the relative size of a deviation is large	1 If the deviation (one or more) fall outside the control limits 2 If the deviations have a certain trend even if they fall in the range of allowable magnitude 3 If the absolute amount of a deviation is financially significant	1. If the probability to revise a standard is high 2. If the cost of uncertainty is large in a d[illegible] cision to investigate based on a priori probability
Basic Requirements Necessary to Exercise Control	1 The establishment of standards based on engineering judgment	1 The knowledge of the $\bar{x}$ and σ of unit cost 2 The ability to determine the range of allowable deviations i e 1 or 2 or 3σ 3 The assumption that past conditions of production will remain the same in the future 4 The distribution of cost is a normal frequency distribution	1 The knowledge of all basic val[illegible] deviations and the expected co[illegible] value not the $\bar{x}$ or σ 2 The ability to determine the a prior[illegible] ability It can be revised [illegible] taining more information 3 The assumption concerning the [illegible] of the manufacturing process so that past conditions remain the same in the future [illegible] not required 4 Normal distribution is not necessary to [illegible] development of the model

Source: Ozel, p 372

Figure 5 Comparative Analysis of Accounting Control Models

value requires the assumption of a normal distribution for the cost.[96] As in the quality control chart technique it is assumed also that the variances are equally likely to be favorable or unfavorable and that they are normally distributed.[97] Thus, there will be

three measures of the desirability of investigation:
1. the absolute size of the variance
2. the size of the variance relative to the size of the standard cost

[Both of these are traditional measures.]

3. the probability of the variance being caused by random non-controllable causes.[98]

The procedure suggested by Bierman, Fouraker and Jaedicke determines the probability distribution of each cost item at every possible level of activity.[99] As in the control chart technique, a range of costs is established to help in the determination of those variances which require investigation.[100] The analyst should also assign weights to the Type I and Type II errors.[101] There are two circumstances in this model which would make it appropriate to investigate a given deviation: either the deviation is deemed likely to occur based on its mean and standard deviation, or its absolute magnitude is so great relative to the firm's financial position that it is significant.[102]

The subjective element of this model lies in the area of the initial

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[96]Ibid., pp. 15-16. [97]Ibid., p. 16. [98]Ibid., p. 18.

[99]Harold Bierman, Jr., Lawrence E. Fouraker and Robert K. Jaedicke, Quantitative Analysis for Business Decisions (Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 111.

[100]Ibid. [101]Ibid., p. 115. [102]Ibid.

setting up of the probabilities of different variances from standard which are then used to calculate the critical probability which is the upper bound of the variances which are felt to be caused from random factors.[103] Figure 6 is a representation of the decision chart and Figure 7, the conditional cost table, used by this model.

A somewhat similar way of looking at the problem is from the point of view of the controllable variances. This view will require four assumptions:

(1) the distribution of possible non controllable cost deviations for each period is normal;
(2) the standard cost is properly set so that the mean of these deviations is zero;
(3) the distribution of possible controllable cost deviations is normal; and
(4) they are independent of the non-controllable cost deviations.[104]

Subjective probabilities are estimated regarding the likelihood or incurring these controllable variances (prior probabilities). The probabilities can, and should, be revised as deviations of a given magnitude are observed (posterior probabilities).[105]

This type of procedure differs from the quality control chart technique in that the "critical points" used to determine the investigation decision area are not spaced equally around zero. This disproportional

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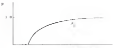
[103]Harold Bierman, Jr., Lawrence E. Fouraker and Robert K. Jaedicke, "A Use of Probability and Statistics in Performance Evaluation," The Accounting Review, XXXVI (July, 1961), p. 412.

[104]Richard M. Duvall, "Rules for Investigating Cost Variances," Management Science, XIII (June, 1967), p. B637.

[105]Ibid., p. B636.



where

p conditional probability of an unfavorable variance uncontrollable causes as large or larger than the actual variance, given an occurrence of unfavorable variance

P_c the critical probability

Rule $p > P_c$ do not investigate

Source Bierman, Topics in Cost Accounting and Decisions, p 18

Figure 6 Cost Control Decision Chart
Unfavorable Variance

States	Acts: Investigate	Acts: Do not investigate	Conditional probabilities of states, given an occurrence of unfavorable variance
1	C	0	p
2	C	L	1 - p
Expected cost of acts	C	L(1 - p)	

Where

C cost of investigation
L present value of the estimate of cost inefficiencies in the future which are avoidable
p probability of State 1 occurring

$P_c = (L - C)/L$

Rule $C < L(1 - p)$ investigate
$C > L(1 - p)$ do not investigate

Source: Bierman, *Topics in Cost Accounting and Decisions*, pp 20-21

Figure 7 Conditional Cost Table

ity occurs for two reasons

1. It is felt that there are greater benefits to be derived if the causes of unfavorable variances are discovered.
2. One of the assumptions of the model is that of positive average controllable deviations; therefore, "the possible negative values of y [the expected deviation] have a small probability of occurrence for a given favorable observed deviation x, relative to the possible positive values of y for an unfavorable observed deviation of the same size."[106]

The Bierman, Fouraker and Jaedicke model may be considered as being more of a transition between the classical statistical and the decision theory model because it continues to be interested in some of the features of the classical models, e.g., the requirement of a normal distribution, the development of a form of control limit, the interest in Type I and Type II errors which are not of primary importance in the decision theory models.[107]

Another model which more closely resembles the modern decision theory form is that proposed by Onsi.[108] This model considers subjective probabilities as comparable to personal judgment.[109] It also

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[106] Ibid., p. B640.

[107] The factors emphasized by a control model which is based on Bayesian analysis have been mentioned by Onsi, p. 324.

[108] Ibid., pp. 321-330. [109] Ibid., p. 325.

begins the analysis by setting up a 'subjective probability distribution for the unknown parameter being investigated.'[110]

Ozsi based his model on two assumptions:

1. The investigation decision is initially based on incomplete information which has been derived from a random sample of the output units; this sample is felt to be "a good representation of the population" from which it is taken.
2. In addition to analyzing the total variance into their price and efficiency components, the accountant also wants to ascertain if the process is stable, i.e.,
   a. Are defective units "controlled within normal expectations?"
   b. Are the standard per unit quantities of material, labor and variable overhead "controlled within the prescribed range?"[111]

This model differs from the classical statistical model in that it looks at the value of information as determined from the "reduction of the expected cost of the proposed initial decision" as compared to the sampling cost. In the classical statistical model the value of information is developed from the "reduction of the magnitude of the standard deviation."[112]

In both the Bierman, Fouraker and Jaedicke and the Ozsi models, the expected cost of each act is used to aid in the investigation decision

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[110] Ibid., p. 324. [111] Ibid., p. 325. [112] Ibid., p. 326.

with the "minimization of expected costs" being used as the decision rule. The major difference between these models is in the method of utilization of the probabilities. In the former model, the analyst is looking at the likelihood of an observed chance deviation being equal to or greater than some "actual probability." The analyst, using the latter model, will be interested in the likelihood of the unknown parameter assuming some specific value.[113]

Controlled Cost Model

This is a method of analyzing cost variances that was suggested by F. S. Luh. The controlled cost system is used to alert management via the traditional management by exception principle to those deviations of actual cost from the controlled cost which require investigation.[114]

The system is based on two assumptions, the first of which is an implicit one:

1. The state of technology has not changed between the time of the determination of the controlled cost and the incurrence of the actual cost.[115]

2. A "distinct probability distribution of cost" exists for the controlled operation.[116]

As in the Osal model, above, this model must rely on incomplete infor-

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[113]Probst, The Utilization of , p. 47.

[114]Luh (Ph. D. Dissertation), p. 42. [115]Ibid. [116]Ibid., p. 90.

mation obtained from a sample of controlled performance This sample is considered as having been taken "from the universe of controlled performance."[117]

The controlled cost as derived from the sample is expressed as a frequency, or distribution, function.[118] It takes the place of the standard cost concept The basic approach of the model is the testing of the hypothesis "that two samples were taken from the same universe."[119] The results of this test are then used in making the investigation decision.[120] The test takes into consideration three variables

1 sample size The size of the two samples being tested
2 precision The magnitude of the difference in the probability distribution of the two samples being tested. . . .
3 reliability The probability corresponding to the precision obtained from tables, i e , the degree of assurance in stating that the two samples being tested are from the same universe.[121]

Figure 8 is a flow chart depicting Luh's procedure In this model the concept of cost deviation must be redefined since the controlled cost has been set up as a probability distribution Thus, "cost deviation is the deviation of the probability distribution of actual cost from the probability distribution of controlled cost."[122] This deviation may be expressed in various ways, depending on the type of distribution function

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[117] Ibid , pp 48-49

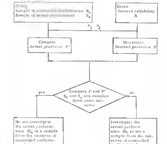
[118] Ibid , p 39

[119] Probst, The Utilization of , p 34

[120] Ibid

[121] Luh (Ph D Dissertation), p 34

[122] Ibid , p 43



Source Luh, (Ph.D. Dissertation), p. 61

Figure 8 Flow Chart of General Test Procedure

being assumed [123]

The use of such a system extends the traditional analysis procedure beyond an analysis of the mean because the cost analysis is based upon a probability distribution. By means of such a more complete analysis of the cost data, previously overlooked deficiencies may be brought to light [124] There are several other ways in which controlled cost differs from the traditional and classical statistical accounting control models [125]

1 The main criterion for measuring the efficiency of performance is a probability distribution — not a single point or a range

2 Cost at all ranges of performance is included, with a probability of occurrence being established at each range

3 Normality is assumed when cost data developed from means of random samples are analyzed by means of theorems regarding sampling distributions of means and variances, but the Kolmogorov Smirnov theorem, which is not tied to any particular distribution, should be used for any other types of cost data [126]

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[123] _Ibid_ , p 90 "For non normally distributed cost the measure is the maximum of the absolute value of the difference between the distribution function of the actual cost and the distribution of controlled cost This measure enables the interpretation of cost deviation in a probability expression by using the Kolmogorov-Smirnov limit theorem Normally distributed cost may be interpreted by comparing means and variances through the use of F-distribution and t-distribution "

[124] _Ibid_ p. 66.

[125] _Ibid._, pp 66-68

[126] See Luh _The Accounting Review_ pp 131-132 for a discussion of the Kolmogorov-Smirnov limit theorem, F-distribution and t distribution as well as references to sources of additional information on these

4. The analysis proceeds on the basis that both the actual costs and the controlled costs are samples from the same universe.

This system is felt to have a number of limitations inherent in it, although some of them are equally applicable to any statistical procedure:

1. The operation being analyzed should be repetitive, at least during the period under analysis.
2. The cost data must be calculated on a frequent basis, e.g., hourly.
3. The establishment of the cost as a probability distribution makes it less suitable for determining product prices than the other systems.[127]
4. As in the control chart approaches, there is no consideration of the costs involved in the investigate/do-not-investigate decision.[128]

Impact on Standard Costs

The preceding three statistical models have carried the concept of a standard cost far from the band of costs concept developed from classical statistics and even farther from the original idea of a benchmark. The "standard" has become an expected value concept or a probability distribution.

Because of the need for the frequent collection of data, all three

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theorems and tables of values.

[127] Luh (Ph.D. Dissertation), pp. 70-71.

[128] Probst, The Utilization of . . ., p. 37.

models must be utilized with repetitive operations This has been considered a limitation in the application of statistical models, but this is not necessarily the case since many operations fit such a mold, particularly the type for which traditional standard costs would be computed

There are a number of similarities between the approach of the control chart models and the models of this section In particular, there is the desire to isolate the controllable deviations for managerial attention The assumption of normality is maintained, although it is no longer a mandatory condition

Modern decision theory models add a new aspect to the investigation decision by looking into the cost of making an investigation This is not considered under the traditional variance analysis system, the classical statistical techniques or the controlled cost procedure but is an important factor in the decision process It can further limit the number of variances requiring managerial attention and yet may include some which normally would be overlooked.

However, these models, just as the traditional and classical statistical models, do not meet all three objectives of a control system [129] The first objective of identifying deviations from current operating efficiency is still met but not the other two disclosure of impending crises and the revelation of means of improving current operating effi-

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[129] See page 77

ciency

The expected cost still is a band of costs, but the range is subjectively determined, based on a sample or past experience, and probabilities are attached to each possible cost to depict the likelihood of their occurrence. As experience with the model is gained the probabilities are revised. This continuous updating of the model gives it a more dynamic character than the traditional or classical statistical models.

Such a system is useful mainly for control purposes; traditional standards would still have to be developed to serve several of the other applications of standard costing.[130] Thus, greater demands will be placed on the cost accountant if any of the statistical models are adopted for use in the analysis of variances.

Summary

This chapter has looked at various statistical control models which are considered to be improvements on the traditional variance analysis procedures. Statistical cost control is based on a number of assumptions, some of which may not be met exactly by the system being studied -- especially the assumption of a normal distribution. All of the models are interested in ascertaining for managerial attention only those variances which it is felt are due to assignable causes; only those deviations, therefore, have the possibility of being eliminated by finding their causes.

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[130] See page 8.

A major difference in the models, besides the type of standard which is developed relates to the making of the investigate/do-not investigate decision Modern decision theory models consider the cost of making the investigation in reaching this decision, the other models do not

Three problems of traditional control techniques are discussed subjectivity in the investigate/do-not-investigate decision, 'average out" problems, and the commission of Type I and Type II errors [131] The statistical models generally help considerably in their solution The investigation decision is no longer solely based on a subjective judgment, and the chance of incurring either a Type I or Type II error is lessened The need for more frequent observations helps relieve the problem of compensating variances Also, significant variances are detected, and corrective measures are carried out, sooner

Central chart techniques provide a number of signals of a possibly out of control process These warnings not only can show when a significant deviation has occurred, but when the process average may have changed thus signalling a need to revise the standard

The major impact of these statistical models on standard costs, however, is the movement from the single point benchmark to a band of standards, an expected value or a probability distribution Despite these changes the statistical models are still only able to signal when

[131] See pages 75 76

a significant deviation from standard has occurred, an after-the-fact datum, and do not provide any before-the-fact indication of trouble spots, although control charts hint at such problems, nor do they give suggestions as to how to improve operating efficiency -- the other objectives of an effective cost control system

V LINEAR PROGRAMMING, OPPORTUNITY COSTING, AND EXPANSION OF THE CONTROL HORIZON

The use of management science techniques for cost control linear programming in particular, is discussed separately in this chapter because the techniques, while still related to standard costing and the traditional concepts of variance analysis, use a different point of view, that of opportunity cost Also, although the techniques are being suggested for use in cost control, their major emphasis is upon planning, with control of only secondary importance

The area of management science also brings up a related topic -- the propriety of the use of standard costs and quantities as data inputs to mathematical programming models [1] This question will be taken up in the second part of the chapter.

Introduction

Several presentations of the traditional variance analysis techniques in a mathematical format exist in the literature [2] The expression of

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[1] The term "mathematical programming" is a more general one which includes linear programming

[2] For example, see Ching wen Kwang and Albert Slavin, "The Simple

these procedures in mathematical terms has two chief advantages

1. There is increased precision in the expression of the techniques; less ambiguity in the meaning of the terms and a clearer exposition of the key elements of the analysis and the computational rule to be followed.

2. Equivalent, alternative formulations are possible, which may be used to reconcile different presentations of the same technique or can help in situations where the data are not available for one formulation, but are for one of the alternatives [3]

Mathematical Programming

Programming techniques meet the above advantages of mathematical formulation. Accountants should find such procedures interesting because of the similarity of the underlying approaches of both accounting and programming to certain managerial problems. Also, accountants will need to supply much of the data used in various managerial decisions in which some sort of programming model will be employed [4]

Mathematics of Variance Analysis," The Accounting Review, XXXVII (July, 1962), pp. 415-432 and Zenon S. Zannetos, "On the Mathematics of Variance Analysis," The Accounting Review, XXXVIII (July, 1963), pp. 528-533

[3]Frank Werner and Rene Manes, "A Standard Cost Application of Matrix Algebra," The Accounting Review, XXXXII (July, 1967), pp. 524-525

[4]Nicholas Dopuch, "Mathematical Programming and Accounting Approaches to Incremental Cost Analysis," The Accounting Review,

The term "mathematical programming" is used here in order to keep the discussion open to the possible introduction, in the future, of any of the programming techniques available -- linear and nonlinear, to the problems of cost control. However two programming techniques are considered useful to the accountant at present, linear algebra and linear programming

> Linear algebra is a computational technique which the accountant may use to develop (1) estimated per unit costs for various products and (2) estimated total activities and inputs required to achieve a given level of net output.[5]

This technique has considerable significance to the problem of service department cost allocation which will be discussed more fully in the next chapter. Linear programming, on the other hand,

> can go a step further than linear algebra in that it can be used in predicting what the desired net output should be. In addition linear programming can handle joint products and multiple sources of inputs, the linear algebra computations cannot handle these situations.[6]

Typically linear programming is employed as a planning device to determine the optimum allocation of scarce resources, an application not generally contemplated for standard cost systems. There is, however, one area of linear programming which may be viewed as a form of variance analysis: parametric programming or sensitivity analy-

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XXXVIII (October 1963), p. 745

[5] Feltham, p. 11.

[6] *Ibid.*

way.[7]

Three basic types of data are employed in linear programming problems: the coefficients used in the objective function, the constraint equations' coefficients and their related constants.[8] Sensitivity analysis is a technique which can be used after an optimal solution has been reached to test the ranges in which those various coefficients may vary without changing the optimal solution.[9] Parametric linear programming leads to "systematic sensitivity analysis;" it is interested in systematically studying the simultaneous changes of a number of parameters of the linear programming model, e. g., simultaneously changing a number of the objective function coefficients.[10] Sensitivity analysis considers changes in only one coefficient at a time.

Opportunity Costing

OPPORTUNITY COST. The maximum alternative earning that might have been obtained if the productive good or service had

[7] Dopuch, p. 752. For a brief discussion of the concepts and limitations of sensitivity analysis see: Joel S. Demski, "Some Considerations in Sensitizing an Optimization Model," The Journal of Industrial Engineering, XIX (September, 1968), pp. 463-466.

[8] Ronald V. Hartley, "Linear Programming: Some Implications for Management Accounting," Management Accounting, LI (November, 1969), p. 48.

[9] Ibid.

[10] Frederick S. Hillier and Gerald J. Lieberman, Introduction to Operations Research (San Francisco: Holden-Day, Inc., 1967), p. 499.

been applied to some alternative product or use.[11]

There is a connection which can be shown to exist between the marginal cost curve of a firm and its standard cost system. The marginal cost curve is needed when standard costing is attempting to measure total variable cost at a given output level -- standard direct costing, and also when it is assigning a cost to variances.[12] Mathematical programming is needed as an aid to marginal costing because of the number of factors to a firm which may be available in limited supply and yet be demanded by several competing uses, e.g., a limited amount of a particular raw material being required by several different products. With only one such factor, marginal costing techniques may be used easily to determine the firm's optimal policy, but it is a more complicated process when there are several such constraining resources. Mathematical programming techniques determine the optimal profit, or minimum cost, subject to several constraints, and in this way the optimal allocation of the scarce resources is determined.[13] "Thus, programming may be viewed as simply a means for extending the advantages of using marginal costing (direct costing) as the basis for short-

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[11] Horngren, p. 568.

[12] Joel S. Demski, "Variance Analysis Using a Constrained Linear Model," in Solomons (1965), p. 528.

[13] J. M. Samuels, "Opportunity Costing: An Application of Mathematical Programming," The Journal of Accounting Research, III (Autumn, 1965), pp. 182-183.

run decisions on price and output."[14] Marginal costing, as the above discussion may be based on either actual variable costs or standard direct costs but, because of the planning aspect of resource allocation the variable costs would tend to be standard costs of some type.[15]

In terms of programming, "variances are regarded as changes in the data inputs."[16] The cost of a "variance" is determined from its effect on the optimum profit and is determined from the shadow prices or opportunity costs developed in the solution. "The resulting opportunity cost figures reflect the effects of variances on net income."[17]

The approach of programming is considered to be *mutatis mutandis*, whereas traditional accounting may be considered to be a *ceteris paribus* approach.[18] The former considers "optimum adjustment" and "gauges significance by determining the opportunities foregone as a result of the deviation and failure to respond to it," while the latter con-

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[14] *Ibid.*, p. 283.

[15] The planning aspect of the model implies a before the fact action which necessitates the use of predetermined costs -- therefore, standard rather than actual. However, it may be possible to incorporate a band of costs into the analysis rather than a single point estimate.

[16] Demski, "Variance Analysis . . . ," p. 526.

[17] *Ibid.*

[18] *Ibid.*, p. 530. *Ceteris paribus* refers to a type of analysis where only one variable is changed at a time while all the others are held constant. *mutatis mutandis* type analysis permits all the variables to be adjusted simultaneously.

-siders the cost of the deviations as being measured only by the difference between the actual and the standard cost at the actual output [19]

To carry out the opportunity costing approach and, in particular, to develop the opportunity costs, it is necessary to determine the optimum, rather than the standard performance at the standard volume which was produced [20] To do this, the traditional analysis has to be expanded to include optimum income [21]

> by analyzing the period's events in terms of their effect on the model inputs and structure, the opportunity cost system provides a framework for introducing the error/performance response decision problem into the accounting process. [22]

Two Suggested Opportunity Cost Approaches

This section will look briefly at the linear programming models proposed by Samuels and Demski and their impact upon standard costing Brief illustrations of these models can be found in the appendices at the end of the study

Samuels' Model

J, M Samuels presented a system in which the shadow prices are incorporated into the responsibility accounting system.[23] These shadow prices as they appear in the optimal solution to the dual programming

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[19] Ibid.   [20] Ibid   [21] Ibid , p 533

[22] Ibid , p. 540   [23] Samuels, p 182

problem (or may be read off the solution to the primal problem) can be used to calculate the opportunity costs. "The shadow prices of the limiting factors reflect the values of their marginal products."[24]

The shadow prices can, under Samuels' system be used as the basis of the standard cost system; they can be employed to charge departments for the use of scarce resources.[25] In the traditional accounting process unabsorbed overhead may result when a department fails to produce its budgeted overhead. The profit which the firm does not receive due to the above failure is considered to be the "real loss" to the firm.[26]

If the department is operating under the optimal plan, it should break even using the shadow prices. It can achieve a profit, favorable variance, it is able to operate better than under the expected technological relationships but "its profit will not be at the expense of one of the other departments."[27] An unfavorable variance, or loss, will occur when the budgeted inputs, as determined from the shadow prices, are exceeded. Appendix C is a summary of Samuels' example of his system.

Samuels feels that his system has some advantages. First, it achieves two objectives: the firm has maximized profits while obtaining a measure of control. Second, no department can suboptimize to meet its own goals irrespective of those of the other departments or the firm.

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[24]Ibid., pp. 183-184.

[25]Ibid., p. 186.

[26]Ibid., p. 185.

[27]Ibid., p. 186.

as a whole without being penalized, i.e., to produce additional output without a penalty, it is necessary that excess capacity, which is priced at zero, be available.[28]

Such a system combines the properties of decision making, as displayed by the marginal costing inputs, with the control features of standard costing, as exercised through the shadow prices which are used to charge the overhead and semi-variable costs to the various departments.[29] These shadow prices act as a replacement for the overhead rates which are usually calculated.

Demski's Model

Joel Demski calls his approach *ex post* analysis. This procedure makes use of two linear programming solutions, the *ex ante* and the *ex post*, and the observed results, and operates under the assumption that the firm has a decision model, or rule, under which it is operating. It is also assumed that the firm periodically revises this model, with the revisions being based on re-estimated data inputs and structural changes.[30]

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[28] *Ibid.*, p. 187. [29] *Ibid.*

[30] Joel S. Demski, *Variance Analysis: An Opportunity Cost Approach with a Linear Programming Application* (Ph.D. Dissertation, University of Chicago, 1967), p. 3.

There are four major assumptions for the *ex post* system:

"(1) that the firm employs some specific well defined formulation of its planning process,

(2) that management possesses the ability to distinguish between avoidable and unavoidable variances or deviations,

(3) that feedback control information is useful, and

(4) that the search for possible opportunities can be limited to

The technique may be considered as being part of the opportunity cost approach because it compares what the firm did accomplish during the planning period being analyzed with what it should have accomplished during the same period.[31]

The *ex post* system, by looking at actual performance and the original plan simultaneously, differs from the traditional accounting system which only views actual performance as it relates to the original plan and, generally, ignores shifts in the latter, i.e., the traditional system looks only at *ex ante* and actual results and the monetary significance of any deviations between these results.[32]

The *ex post* analysis goes one step further than the traditional system. It recomputes the optimal plan, as calculated in the *ex ante* program, using the observed figures to re-estimate the inputs. The new solution represents the optimum program that should have been determined if the initially forecasted data had been correct.[33]

Traditional variance analysis views the difference between actual and standard results for a specified output; *ex post* analysis, in contrast also implicitly signals output deviations and develops opportunity costs

scope ... to the existing planning model." Joel S. Demski, "An Accounting System Structured as a Linear Programming Model," *The Accounting Review*, XXXXII (October, 1967), p. 702.

[31] Demski, *Variance Analysis* ..., p. 2. [32] *Ibid.*, p. 3.

[33] *Ibid.*

for all deviations.[34] Thus, there are two important differences between the ex post and the traditional accounting systems:

1. The comparison is between actual and ex post optimum results, not between actual and ex post or ex ante standard results at a given output, i. e., output is considered as an endogenous variable for ex post systems, while it is treated as an exogenous variable in the traditional variance analysis techniques.
2. The analysis covers all planning model inputs, not just the factors which show up in the optimal solution, i. e., cost and/or revenue factors.[35]

The results which are obtained and the meaning of their differences may be summarized as follows:

> . . . three sets of results: the ex ante, the observed, and the ex post. The difference between ex ante and ex post results is a crude measure of the firm's forecasting ability. It is the difference between what the firm planned to do during the particular period and what it should have planned to do during the particular period. Similarly, the difference between ex post and observed results is the difference between what the firm should have accomplished during the period and what it actually did accomplish. It is the opportunity cost to the firm of not using its fixed facilities to maximum advantage. Specifically, it is the opportunity cost of non-optimal capacity utilization.[36]

Appendix D is a brief summary of Demski's mathematics and two examples of how his method might be applied.

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[34] Ibid., p. 4. [35] Ibid., p. 22.

[36] Demski, "An Accounting System . . . ," p. 702.

There are three main reasons which may be cited as to why the traditional techniques used in variance analysis may fail to signal changes in the factors which are involved in the firm's output decision:

1. The standard cost system, as normally conceived, often does not contain such factors, e.g., selling prices, prices of possible substitute materials.
2. Measurement errors may have occurred, thus causing some changes to be ignored or included inaccurately in the analysis.
3. Changes in the underlying distribution of some of the factors may be difficult, or impossible, to determine because of their stochastic nature.[37]

Ex post analysis is felt to be better than traditional variance analysis because of the additional information it makes available to management:

1. It shows "the best that might have been done" under actual conditions prevailing in the period under analysis.
2. The "exact source of each perturbation" is established based upon both the inputs to and the structure of the model.[38] In addition, an estimate of the "associated opportunity cost signifi-

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[37] Demski, *Variance Analysis*, pp. 29, 31.

[38] *Ibid.*, p. 23. "Perturbation" refers "to any deviation or change in the data inputs or structure of the firm's planning model -- that is, any prediction error, control failure, model error et cetera. . . . a perturbation is separate and distinct from a variance; variance refers to the dollar effect of some deviation from standard. In other words,

cance" is provided,

3 A given perturbation is felt to have an effect upon other responsibility centers and this effect is shown [39]

In contrast to this, the traditional analysis

1 looks merely at comparisons of ex ante standard results with actual results, the standard results are not revised in the light of actual conditions

2 observes the source of the perturbations and the significance of the opportunity costs,

3 assumes that the responsibility centers are completely independent of one another [40]

Impact of Opportunity Cost Concept Models Upon Standard Costing

Both of the foregoing models use the concept of traditional variance analysis as their starting point, but they go much beyond such techniques. The costs which are analyzed are no longer standard in the traditional sense. The main relationship to standard costing is the use of the opportunity cost models for control purposes. These models also are based on the concepts which make up direct costing, or direct standard costing, in that they are concerned only with the variable costs of the firm.

The variances which are analyzed relate to changes in the data inputs

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perturbations cause variances "

[39] Ibid., pp. 94-95.

[40] Ibid.

to the models When thinking in terms of the cost coefficients for the constraint equations, there may be some similarity to the traditional standard costs, but they are not the same thing This will be discussed more fully in the section on data inputs to programming models.[41]

Because programming models have as an objective the optimization of some "figure of merit," usually the maximization of income the variance analysis hinges on the effect of changes in the data inputs on income This concept probably is implicit in traditional variance analysis, since unfavorable variances do act to reduce income Because of the shadow prices, which are developed as the primal problem is solved, the cost of the input changes can be determined and, if carried further through the use of sensitivity analysis or parametric programming, it is possible to determine the ranges within which the coefficients may vary before the existing solution is no longer optimal. In this way single-value costs need not be binding on the analyst and may be replaced by a range Also, through sensitivity analysis it is possible to determine which inputs are critical to the solution and thus, should be estimated with greater precision than the less critical ones

Among the advantages of the ex post system is that it is able to take account of factors not normally considered in traditional standard cost models i e , selling prices, prices of substitute materials Solomons mentions five elements which may make up the material price variance

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[41]See pages 133-137.

but which are not usually analyzed separately

1 The result of price fluctuations which have occurred since the standards were set
2 The result of inefficient buying
3 The result of substitutions differing from standard
4 The result of inflationary pressures on prices in general
5 The effect on the buying price of purchasing in more or less than the budgeted quantities [42]

The third factor often is analyzed separately through "mix" variances, but the others may be ignored by the traditional procedures If the programming model is used, the possibility of the use of substitute material can be included in the model directly Opportunity costs for these materials not in the optimal solution are provided, thus making the analysis of the effect on income from using one of the alternative easier Some of the other price elements may be looked into also through the use of sensitivity analysis.

Data Inputs to Programming Models

Several authors have argued that standard costs, as well as historical costs, are not appropriate for use as inputs to the various types of linear programming models [43] They recommend the use of opportunity

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[42]David Solomons, "Standard Costing Needs Better Variances," N A A Bulletin XXXXIII (December, 1969), p 32.

[43]A Charnes, W W Cooper, Donald Farr and Staff "Linear Programming and Profit Preference Scheduling for a Manufacturing Firm," in Analysis of Industrial Operations, Eds Edward H Bowman and Robert B Fetter (Homewood Ill. Richard D Irwin Inc., 1959), p 32. Howard Gordon Jensen, Some Implications of the Cost Data Requirements of Linear Programming Analysis for Cost Accounting (Ph D Dissertation, University of Minnesota, 1963), p 32

costs.[44] "Foregone benefits as well as actual outlays need to be simultaneously considered in the programming process."[45] Despite this strong feeling, expressed by Operations Research men in particular, it is also conceded that the costs derived under a traditional cost accounting system have utility in arriving at estimates of the appropriate costs.[46] Especially where the cost accounting system has been set up as a responsibility accounting system will the initial estimates of variable overhead be dependent upon the standard quantities of labor or machine hours, for example, for each of the several production departments.[47]

Management science techniques require the estimation of "two external 'quasi cost' categories" in addition to the normal accounting costs. The most important of these "is the potential income . . . which the capital invested in the business could earn if invested elsewhere [an opportunity cost]." The other quasi cost has to do with sales

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[44] A recent exception to this belief is presented in Richard B. Lea, "A Note on the Definition of Cost Coefficients in a Linear Programming Model," *The Accounting Review*, XXXXVII (April, 1972), pp. 346-350. Lea compares the opportunity costs to "exit" prices, the use of which he feels is contrary to the concept of a going concern. To reflect the going concern view the costs used in linear programming models should be "entry" prices. However, the choice of the costs to be used in a planning model should depend upon the time period of the plan: short run (exit prices) or long run (entry prices).

[45] Charnes, Cooper, Farr and Staff, p. 32.

[46] *Ibid.*; H. G. Jensen, p. 22. [47] H. G. Jensen, p. 104.

revenue."[48] Management science also deals mainly with costs which are viewed in relation to "specific courses of action and specific assumptions."[49] This is not true of accounting which looks at absolute standards and costs.[50]

Linear Programming Model Coefficients

There are two sets of coefficients which need to be determined for linear programming models: those representing the values of the various activities (objective function coefficients) and those depicting the technical requirements of the activities (constraint equation coefficients).[51] There also is a set of constants which relate to resource availabilities in the firm (e.g., floor space, total available labor hours, power plant capacity).

These three groups of parameters are predictions, especially as initially determined.[52] As such there are four properties which must

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[48]Fred Hanssmann, _Operations Research in Production and Inventory Control_ (New York: John Wiley and Sons, Inc., 1962), p. 79. The second quasi cost may be explained as follows: "If the system can be operated in two modes, A and B, where mode A results in a lower sales volume, then there is an opportunity cost of mode A relative to mode B equal to the marginal profit differential between the two modes . . . the profit differential must be calculated exclusive of the cost differential attributable to a change from mode A to mode B."

[49]_Ibid._, p. 80. [50]_Ibid._, p. 79.

[51]H. G. Jones, pp. 18-19.

[52]Richard B. Lea, "Estimating the Parameters in Operational Decision Models: A Linear Programming Illustration" (Working paper 71-50, The University of Texas at Austin, May, 1971), p. 4.

1. Variability -- because of the deterministic nature of linear programming models, any parameter which may be variable can be represented by only one value; part of the problem is deciding which one to use which will depend, in turn, upon the objectives of the program.

2. Accuracy -- although accuracy is desirable, it should be weighed against the increased cost which would be incurred to achieve it.

3. Relevant range -- there is a band of values over which the various coefficients may be valid for the problem. These may be predicted when the model is set up; this necessitates the anticipation of the optimal solution, which must fall within the range, and then testing the prediction after the solution is reached.[53]

4. Standards of performance -- several interpretations of this property are possible, including:

   1) Standards attainable given good performance and use of proper methods.
   2) Standards which require a high degree of performance or achievement and are likely unattainable on any sustained basis.
   3) Standards which are so easily conformable that a significant amount of unavoidable waste and inefficiency is accommodated by the standard.[54]

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[53] Ibid., pp. 5-6, 8.

[54] Eric Kohler, A Dictionary for Accountants (2nd ed.; Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957), p. 452 as quoted by Lee, "Estimating the Parameters . . . ," p. 9.

Of these costs the [illegible] concept is appropriate for linear programming.[55]

"The technical coefficients are estimates of the quantities of the restraining factors which will be utilized by one unit of the activity or product."[56] The inputs to be employed must be those whose usage varies directly and proportionately with production. Thus, any input affected by the learning process cannot be included because it is employed in a decreasing proportion to the increased output.[57] Such data are generally determined by engineers but, if the firm has a standard cost system, they may be established from the standard product specifications. If neither of the above type of estimates are available, past accounting records may be used.[58] Regardless of the type of estimate employed, it should be regarded only as an initial valuation which will be tested and revised as the linear programming model is used.[59] The objective function coefficients, which may be made up of net revenue, variable costs or selling prices, will be the ones most affected by the cost estimates.[60]

This will have an important implication for the cost accountant. The accounting system should be set up so "as to collect data on the activities which can be used to test the technical coefficients [which were]

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[55] *Ibid.*, p. 9 [56] M. G. Jensen, p. 15

[57] Lea, "Estimating the Parameters . . . ," p. 11

[58] M. G. Jensen, pp. 19-20 [59] *Ibid.*, p. 20. [60] *Ibid.*

started with a process similar to the calculation of the labor and material variances.[61] The difference between the traditional accounting cost data collection process and that necessary for linear programming lies in the need to more closely scrutinize the services actually flowing into a product.[62] Standard costing operates most unambiguously in the area of production department costs, but the service department costs making up part of the variable overhead are not handled as a "service flow" and thus the system breaks down in its usefulness at this point.[63]

Opportunity cost, especially as related to outlays, should be of interest to accountants.

> The product of an activity results from the injection of productive services in fixed ratios into the activity. Thus, the opportunity cost of an activity or a product is equivalent to the opportunity cost of the productive services flowing into the activity . . . the opportunity cost of an activity is the largest value that the productive services needed to produce that activity at unit level would yield in their best alternative use.[64]

Required Changes in Standards

There are four basic elements which are applicable to all the costs being developed as linear programming inputs.

(1) While the technical coefficients and the constants associated with the restriction equations are within the province of engineering and marketing, adequately detailed records either on a standard cost basis or on an actual cost basis will be helpful in their estimation.
(2) The cost coefficients require an opportunity cost orientation.

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[61] *Ibid.*

[62] *Ibid.*

[63] *Ibid.*

[64] *Ibid.*, pp. 22-23.

of the accounting system

(3) All of the data -- constants and coefficients -- need not meet absolute accuracy standards

(4) The accounting system should be designed to reflect the activities being programmed If there are direct nonvariable costs associated with each activity, these should be identified in the system [65]

<u>Changes in material standards</u>

The direct material standard cost is usually set to reflect "the cost at the level of optimum attainable efficiency"[66] The standard quantity generally is determined from engineering studies and may include an allowance for expected waste and various other losses This quantity standard usually has an incentive motive behind its construction which will lead to frequent incurrences of unfavorable variances.[67] If such a quantity estimate is to be used in a linear programming model, it would need to be adjusted to take into account the expected unfavorable variances [68]

The standard material price generally is established at the price which is expected to prevail during a given period Partial allowances may be made for things such as a standard scrap value before the final standard material price is established for the product,[69] but the standard price may fail to consider the effect of order costs or quantity discounts, for example Thus, the standard material price 'leaves some-

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[65] Ibid , pp 29-30

[66] Ibid , p 54

[67] Ibid , p 55

[68] Ibid pp 55-56

[69] Ibid , p 56

thing to be desired for linear programming analysis," especially in the area of the estimation of variable acquisition costs.[70]

<u>Changes in labor standards</u>

The standard labor time for a product is generally composed of the expected time plus various allowances, e.g., fatigue, unavoidable delays, with the added factor of an incentive for improvement. As in the material quantity standards, unfavorable variances will predominate and this tendency should be taken into consideration in the construction of the linear programming equivalent.[71]

The standard labor rate may also require adjustment for linear programming usage in order to take into account various significant "fringe benefits," such as payroll taxes, allowances for vacation pay or workmen's compensation which may not be considered part of the traditional standard labor cost, although they may be included as part of the variable overhead costs.[72]

<u>Changes in overhead rates</u>

Variable overhead inputs generally are not calculated on a quantity basis. Such quantities, as related to activity levels, may be determined by statistical analysis of historical data, but three problems may arise in such predictions:

1. Existing accounting records generally show only the monetary

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[70]*Ibid.* [71]*Ibid.*, pp. 73-74. [72]*Ibid.*, pp. 74-75.

ude of these inputs, not the quantities, and these latter figures may not be available

2. The data may be accumulated on a departmental rather than a product basis.

3. The true causal relationship between variable overhead input quantity usage and activity levels may be unknown thus requiring more care in predicting the relevant range for these inputs, especially since linear programming requires the use of a linear function despite the actual relationship.[73]

Standard variable overhead rates are ordinarily determined from budgets and are usually related to other quantity standards, e.g., direct labor hours. In a standard cost system, the budget is most likely to be made up of standards for a number of diverse items, fixed and variable, and represents "costs that would be incurred if standard performance were equalled."[74] The analyst should be aware of two things: the development of "full" product costs"; such costs are unsuitable for linear programming coefficients; and second, the cost basis of the budget being used — standard or "incurred" (expected). If the budget is based on standard cost, the variable items should be converted to an expected cost basis.[75] Also, when including these costs in a linear programming model, the effect of any change which has been made in the standard

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[73]Lea, "Estimating the Parameters . . . ," pp. 11-12.

[74]H. G. Jones, p. 92.

[75]*Ibid.*, pp. 91-92.

labor hours for example should be taken into account in the overhead rates based upon the hours

General changes needed in the collection of data

There are a number of changes which linear programming models necessitate in the collection of data, some of which, if implemented might also improve traditional standard costing [76]

1. Transactions data should not be the primary means of obtaining data since such data generally are unable to provide 'current estimates for all model parameters.'[77]
2. Data on nonmonetary aspects of inputs should be made available e.g., quantity data for variable overhead
3. Data should be collected on the current input and output limitations
4. Data should be available currently on products and processes not involved in the current planning period
5. The data should be assembled so as to reflect their variability which will help in establishing the degree of accuracy needed in the more sensitive parameters.
6. The time interval of the data collection should be changed from those of the traditional calendar periods to intervals which lie

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[76] Lee, "Estimating the Parameters . . . , pp 25-27

[77] *Ibid* , p 25

within the planning period used by the linear programming model.

The even numbered changes could bring about improvements in traditional standard costing and variance analysis. Collection of variable overhead quantity data could help in providing more meaningful overhead variances and better control over the related costs; responsibility for variances might be more closely ascertained.

The current collection of data on products and processes not presently used might aid in situations where, for some reason, the products or processes determined by the optimal program can no longer be used. Data on alternatives may help in determining which is the best of the available alternatives to substitute and, thus, again help in the area of cost control.

Finally, the concept of the calendar period has been criticized as being artificial and unrelated to any planning period concept. It has been brought out in the statistical models of Chapter IV that the more frequent data collection and analysis, e.g., hourly, daily or weekly, provides better control over costs. Also, if total costs are accumulated over the entire planning period, which may be more or less than a calendar year (or twelve-month period), a better concept of the costs and deviations may be determined for the project.

Effect on uses of standard costs[78]

The emphasis in this section has changed from cost control to planning. When using standards for the purpose of control or the evaluation of performance, it is necessary to set them as tight as is felt to be possible of attainment by workers because of the use of wage incentives to gain better performance. If the standards are to be utilized for planning or inventory costing, it is more appropriate to make them more realistic, since they will be involved in future decision making of the firm or in income determination.

The changes in the variable cost standards necessitated by linear programming will help in the planning or inventory costing function by reducing or eliminating the tightness factor built into the quantity standards thus following the first concept of the standards of performance mentioned on page 131. They also make the cost accountant more aware of the different factors which may affect the costs and, therefore, should be included in the standard cost.

Summary

This chapter and the preceding one have been concerned with cost control. Two different, but related, topics have been taken up in this chapter. The difference relates to the use of standards being considered control versus planning. Their similarity lies in the linear program-

[78]See page 8 for a list of possible uses of standard costing.

ming orientation utilized. The first section discusses the use of linear programming models and the optimum solutions, in particular, with no consideration of the types of inputs employed. The question of the type of input is studied in the second section. The validity of the results obtained in the first instance are highly dependent upon the model inputs.

The opportunity cost models of Samuels and Demski have changed the traditional concepts of variance analysis by working from optimum planning solutions which necessitates the inclusion of an additional factor in the analysis: income. Samuels' model differs from Demski's in that it uses the opportunity costs as transfer prices and the optimum solution as the budget. Any department operating at other than the budgeted amount is to be charged for the excess usage of the scarce resources. Demski, in his ex post analysis, breaks the difference between ex ante and observed net income created by an unavoidable perturbation into the summation of two differences: the first, ex ante net income less ex post net income, represents the variance due to forecasting error; the second, ex post net income less observed income, is the opportunity cost incurred by ignoring the perturbation. The summation of these differences equals the variance which would be obtained under the traditional standard cost variance analysis techniques.

Traditional standard costs, although they can be used as initial data inputs to a linear programming model, should be subjected to some revisions to improve their utility as data inputs -- the tightness of the

quantity standard should be eliminated or taken into account in some fashion. The factors entering into the establishment of the price standards should be analyzed to make sure everything of consequence has been included. The need to more carefully look into the type and cost of services flowing into a product may help to improve the standard costs, particularly those relating to the variable overhead rate which is presently established in a somewhat ambiguous fashion. Improving standards for planning purposes should lead to better standards for their other uses, i.e., control, inventory costing, evaluation of performance, price setting.

[illegible] ALLOCATION OF COSTS

Two types of cost allocation are possible, both of which are indirectly related to standard costing. One involves the allocation of joint costs existing at the split-off point between the separate products resulting from a joint productive operation, e.g., two or more co-products. The other type of allocation occurs in a manufacturing firm made up of producing departments and two or more service departments; the costs of the service departments must be charged, on the basis of predetermined allocation percentages, to the producing departments.

Both of these allocation problems will be discussed in this chapter after a brief general discussion of the concepts of allocation. The two problems will be looked at in terms of the traditional methods which have been used, proposed improvements, and the impact of the improvements on standard costing.

Introduction

"Cost allocation consists of taking costs as accumulated and further dividing and recombining them to achieve the desired type of

'cost.'"[1] The first step to be performed when analyzing costs is "the measurement of benefits to be derived from the cost or expense elements which are not clearly identifiable with specific departments or cost centers."[2] The accuracy of the determination of these interdepartmental relationships will affect the reliability which may be attached to the allocations which follow in the future.[3] The second step is the actual distribution of the costs based on the allocation ratios.[4]

There are at least three basic ways in which costs may be assigned, all of which may be used concurrently within a firm, department, or cost center:

1. direct application: this approach is valid only when it can be shown that there is a "demonstrable and immediate relationship" existing between the cost and the thing it is being assigned to.

2. allocation: this technique is used in situations where the relationship between the cost and the thing it is being applied to is demonstrable but not direct and precisely ascertainable;

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[1]Langford Wheaton Smith, Jr., *An Approach to Costing Joint Production Based on Mathematical Programming with an Example from Petroleum Refining* (Ph.D. Dissertation, Stanford University, 1962), p. 10.

[2]Thomas H. Williams and Charles H. Griffin, "Matrix Theory and Cost Allocation," in *Management Information: A Quantitative Accent*, Eds. Thomas H. Williams and Charles H. Griffin (Homewood, Ill.: Richard D. Irwin, Inc., 1967), p. 134.

[3]*Ibid.* [4]*Ibid.*

3 proration: this procedure is employed when costs must be assigned to things to which they bear no demonstrable relationship.[5]

Cost control is one of the primary objectives of standard costing. For the control mechanism to be effective, costs should be identified with responsibility centers,[6] and, in turn, charged to the supervisor who exercises control over the costs.[7] Many traditional allocation systems prorate burden costs over the productive departments which may not be, according to some authors, the appropriate form of distribution.[8] Others feel these costs should be assigned to products because such an allocation "makes possible a clearer picture of the relative strength of different segments of business (in this case, products) and the areas where improvement is needed."[9]

Cost allocation, however, should not be considered one of the primary tools of cost control.[10] For purposes of cost control, the allocation and proration techniques are inconsistent with the basic precept of cost control: "to gather costs on homogeneous packages of responsibility."[11] These techniques are useful, however, for purposes of pricing and profit measurement.[12]

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[5]John A. Beckett, "A Study of the Principles of Allocating Costs," The Accounting Review, XXVI (July, 1951), p. 327.

[6]Williams and Griffin, p. 134.

[7]Ibid., p. 29.

[8]Ibid.; Gordon, p. 576.

[9]Beckett, p. 330.

[10]Ibid., p. 329.

[11]Ibid., p. 333.

[12]Ibid.

Service Department Cost Allocation

Service departments are those units in a manufacturing firm which exist to provide aid to the production cost centers; some examples are maintenance, power, personnel and the storeroom. These departments despite their diverse functions possess several characteristics in common:

(1) It is difficult to establish a meaningful measure of their production.
(2) A given level of the firm's output can be realized with various levels of service department activity measured quantitatively by the cost incurred.
(3) service department costs cannot be made to change rapidly without serious indirect consequences.[13]

If such departments only served the production units, there would be no problems insofar as allocating their costs, but they also serve each other, in many cases, which gives rise to the problems involved in the making of reciprocal allocations.

This section will first discuss the traditional procedures used in allocating service department costs where reciprocal relationships exist. Included will be some suggestions by G. Charter Harrison. Then the application of the technique of matrix algebra to the problems created by reciprocal relationships will be taken up, along with a specific example of how such a technique may be employed. The use of input-output analysis, although a subset of the matrix algebra approach, will be taken up in a separate section because of the specific assumptions

[13] Gordon, p. 580.

and uses. The impact of the matrix algebra techniques on standard costing will also be discussed.

Traditional Allocation Techniques

There are several basic methods of service department cost allocations which have been used, or suggested for use:

1. "[Distribute] the expenses to the various departments on a load or service basis, and in turn the expense is redistributed to the other producing departments."[14]

2. First separate expenses into a fixed and variable classification; distribute fixed costs "on a use or demand for service basis"; distribute variable costs "on the basis of actual use of that service."[15]

3. Allocate "the service department expenses directly to the producing departments."[16]

The first two methods have a "pyramiding effect" whereas the latter one avoids it. Any standard cost used in variance analysis for these departments should be "the standard cost of the actual consumption," not the standard cost of standard consumption.[17]

Harrison advocated the last of the allocation techniques, above, and distributed service department costs solely on the basis of a machine

[14]Upchurch, p. 73.

[15]*Ibid.*, pp. 73-74.

[16]*Ibid.*, p. 74.

[17]*Ibid.*

rate He justified this as follows

> There is nothing new in the use of machine rates as a medium of burden distribution but it is a somewhat remarkable fact that apparently the leading exponents of their use have not realized that in machine rates they have in their grasp the means of bringing cost accounting into line with modern industrial thought as expressed in scientific management methods So completely has the accounting mind been obsessed by the idea that the sole object of cost accounting is to distribute expenses in such a manner as to obtain correct information as to the costs of manufacture that the fact that in machine rates we have the ideal vehicle for furnishing operating efficiency data does not seem to have been realized A machine rate is a standard cost and a comparison of the machine earnings and the cost of operating the machines provides the simplest and most effective means of furnishing efficiency data The advantage gained from the use of machine rates as a medium of expense distribution though important is not to be compared with that resulting from their use as a means of comparing the actual expense with the standard [18]

This type of process, i.e., ignoring any type of reciprocal relationships, is the one which may be found in many textbook discussions of the allocation of service department costs [19]

Once the possibility of reciprocal relationships is acknowledged however there are two methods of allocation which have been suggested The first of these uses successive iterations and is almost a trial and error procedure. The other scheme uses simultaneous equations [20] 19

[18]G Charter Harrison, Cost Accounting as an Aid to Production (New York: The Engineering Magazine Co 1924) p 106, as quoted by Upchurch p. 75.

[19]For example, see Henrici, Chapter 10. Henrici uses a sold-hour rate as a standard selling price charged to the using departments for services rendered

[20]Williams and Griffin pp 135-136

In the first method of successive iterations the cost to each service department is distributed as if it were the final distribution. Then these new estimates are again distributed. This process of distributing prior estimates to arrive at new estimates stops when there is stability in the account balances.[21]

The simultaneous equation method uses a series of linear equations. To set up such a system it must be assumed "that the total charges to any department ... shall be the sum of the direct charges to that department, plus a specified fraction of the total charges of each of the other departments."[22] For example, assume a firm has three service departments, A, B, and C, with direct charges D_A, D_B, and D_C respectively. The total charge, T, for each department may be expressed in the following set of equations where P_{ij} represents the allocation percentage from department i to department j.[23]

$$T_A = D_A + P_{AB}T_B + P_{AC}T_C$$
$$T_B = P_{BA}T_A + D_B + P_{BC}T_C$$
$$T_C = P_{CA}T_A + P_{CB}T_B + D_C$$

As long as the number of equations and unknowns is not too large,

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[21] Ibid., p. 136; Williams and Griffin, The Mathematical ..., p. 98.

[22] Cuthbert C. Hurd, "Computing in Management Science," Management Science, 1 (January, 1955), p. 108.

[23] Ibid.

[illegible] solved algebraically after rewriting it as follows

$$D_A = \quad T_A - P_{AB}T_B - P_{AC}T_C$$
$$D_B = -P_{BA}T_A + \quad T_B - P_{BC}T_C$$
$$D_C = P_{CA}T_A - P_{CB}T_B + \quad T_C$$

If the system is very large and cumbersome the next logical step is to move to matrix algebra since the algebraic solution of simultaneous equations uses many of the principles involved in matrix algebra theory [24]

Matrix (Linear) Algebra

Linear algebra is particularly useful in the allocation of service department costs where (1) there are reciprocal relationships, (2) a large number of department, and (3) the ability to express the relationship as a system of simultaneous equations, as in the preceding section [25] "Matrix algebra provides a systematic theory for systems of m equations and n unknowns It explains the conditions under which such systems will have no solution, a unique solution or infinitely many solutions ." [26] In the problems under discussion here, the system will have a unique solution because there will be n equations in n unknowns a square matrix, and many of the properties necessary

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[24] Williams and Griffin "Matrix Theory ," p 136

[25] Ibid., p 146 Williams and Griffin, The Mathematical p 101

[26] Williams and Griffin, The Mathematical , pp 146 147

of a unique solution are also assumed to hold.[27] A basic assumption of such computations is that the user knows the net output which is one characteristic which makes this a different approach than linear programming which may be used to calculate the desired final output.[28]

If the example of the preceding section is recast in matrix notation it would appear as follows

$$\begin{bmatrix} 1 & -P_{AB} & -P_{AC} \\ P_{BA} & 1 & -P_{BC} \\ -P_{CA} & -P_{CB} & 1 \end{bmatrix} \times \begin{bmatrix} T_A \\ T_B \\ T_C \end{bmatrix} = \begin{bmatrix} D_A \\ D_B \\ D_C \end{bmatrix}$$

If the first matrix on the left, which shows the distribution coefficients, is called A, the vector of unknowns, X, and the vector of the costs to be distributed, B, the system may be expressed as $AX = B$.

An important by-product of the matrix algebra calculations is the inverse, A^{-1}. This inverse arises when the system $AX = B$ is solved for X: $X = A^{-1}B$. This new matrix does not change once it is determined unless there is a change in some of the elements which made up the original allocation percentages matrix, A; it is "permanent." This property is very useful since the same inverse may be used for later cost allocations thus necessitating only a matrix multiplication, $A^{-1}B$,

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[27] For a discussion of the properties which a matrix must have in order to derive its inverse, and ensure a unique solution, see, for example, George B. Dantzig, Linear Programming and Extensions (Princeton, N. J.: Princeton University Press, 1963), pp. 189-195.

[28] Feltham, p. 20.

[illegible] new X figures for each period in which B changes.[29]

Illustration

The example to be described below has been used by several authors.[30] The company has five service departments and three manufacturing departments. The following allocation percentages have been developed for the amount of service provided to each of the various departments.

From Service Department	1	2	3	4	5
To Service Department					
1	0	0	5	10	20
2	0	0	10	5	20
3	10	10	0	5	20
4	5	0	10	0	20
5	10	10	5	0	0
To Manufacturing Department					
A	25	80	20	0	10
B	25	0	30	40	5
C	25	0	20	40	5
Total	100	100	100	100	100

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[29]Williams and Griffin, "Matrix Theory . . . ," p. 142; Williams and Griffin, The Mathematical . . . , p. 100.

[30]For example, Williams and Griffin, "Matrix Theory . . . ," pp. 140-141 and John Leslie Livingstone, "Input-Output Analysis for Cost Accounting Planning and Control," The Accounting Review, XXXXIV (January, 1969), pp. 48-49.

The service department costs to be allocated are:

Department	Cost
1	$ 8,000
2	12,000
3	6,000
4	11,000
5	13,000

In terms of simultaneous equations the problem may be set up as follows

$$X_1 = 8,000 + 05\,X_3 + 10\,X_4 + 20X_5$$
$$X_2 = 12,000 + 10\,X_3 + 05\,X_4 + .20X_5$$
$$X_5 = 6,000 + 10\,X_1 + 10\,X_2 + 05X_4 + 20X_5$$
$$X_4 = 11,000 + 05\,X_1 + 10\,X_3 + 20X_5$$
$$X_5 = 13,000 + 10\,X_1 + 10\,X_2 + .05X_3$$

where X_i (i = 1 , 5) represents the total service department costs after all the reciprocal distributions have been made. This system may be rewritten as follows

$$\begin{array}{rrrrrcr} X_1 & & -\ 05X_3 & -\ 10X_4 & -\ 20X_5 & = & 8,000 \\ & X_2 & -\ 10X_3 & -\ 05X_4 & -\ 20X_5 & = & 12,000 \\ -\ 10X_1 & 10X_2 & +\ X_3 & 05X_4 & -\ 20X_5 & = & 6,000 \\ -\ 05X_1 & & -\ 10X_5 & +\ X_4 & -\ 20X_5 & = & 11,000 \\ 10X_1 & -\ 10X_2 & 05X_3 & & +\ X_5 & = & 13,000 \end{array}$$

This system is solvable in the present form, but as a way of avoiding such a lengthy process it may be re-expressed in a matrix format

$$\underbrace{\begin{bmatrix} 1 & 0 & -.05 & -.10 & -.20 \\ 0 & 1 & -.10 & -.05 & -.20 \\ -.10 & -.10 & 1 & -.05 & -.20 \\ -.05 & 0 & -.10 & 1 & -.20 \\ -.10 & -.10 & -.05 & 0 & 1 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 8{,}000 \\ 12{,}000 \\ 6{,}000 \\ 11{,}000 \\ 13{,}000 \end{bmatrix}}_{B}$$

In equation form this would become $AX = B$. Since it is necessary to determine X, we must first derive A^{-1}, the inverse of matrix A. This may be done by a computer program. The formula to be worked with one the inverse is obtained is $X = A^{-1}B$ and, thus, X can be determined by a simple matrix multiplication so long as the percentages used in A do not change. This operation will give the redistributed cost of the service departments after all service department costs have been allocated internally.

The allocation of the service department costs to the manufacturing departments will be carried out by another matrix multiplication using the matrix of service department allocation percentages to the operating departments and the X_i's determined in the first operation to arrive at the total service department costs T_r (r = A, B, C) to be added to the other manufacturing costs of each producing department. Thus, from the data on page 151, the operation would be written as:

$$\begin{bmatrix} 25 & 80 & 20 & 0 & 10 \\ 25 & 0 & 30 & 40 & 05 \\ 25 & 0 & 20 & 40 & 05 \end{bmatrix} \times \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

Impact on Standard Costing

The chief impact of matrix algebra techniques on standard costing is to facilitate the calculation of the service department overhead to be added to each service department's costs and then to the producing departments' costs. This is especially true after the initial application of the process which derives the inverse of the matrix of allocation percentages. This overhead will be used in the variance analysis for the departments involved. The technique, however, does nothing to ensure the appropriateness of the allocation percentages or the reliability of the costs being allocated.

A possible drawback of the technique is the need for a computer to arrive at the inverse, especially of the matrix A is very large. Once the inverse is obtained, however, the product $A^{-1}B$ may, if necessary, be carried out by the use of a calculator.

Input-Output Analysis

This is a technique borrowed from the area of macro-economics. In its economic context "the input-output model . . . analyzes transactions between economic activities where activities generally are

ment) at industr[illegible] in terms of smaller units, i.e., a firm, a department or a cost center.[31] The model, originated by Wassily Leontief, displays a summary of all transactions between the economic entities being analyzed in the format of a square matrix.[32]

The General Model and Its Assumptions

The basic model assumes that there is only one primary input to and output from each activity. Each of these outputs may be a final product, or an intermediate product which is used as an input to other activities.[33] There are two possible ways of viewing such a system "each leading to a different concept of economic activity."[34]

1. Output-oriented systems: in this concept, the outputs are known and the inputs must be determined. This is the more common format.

2. Input-oriented systems: in this format the inputs are given and the outputs are unknown; this system is less common but will be used in some of the standard cost accounting applications to be discussed below.

Each of these systems is bound by the same set of basic assumptions which are applicable to any linear algebra or linear programming model

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[31] Livingstone, p. 51. [32] Ibid., p. 50. [33] Ibid., p. 51.

[34] John E. Butterworth and Berndt A. Sigloch, "A Generalized Multistage Input-Output Model and Some Derived Equivalent Systems," The Accounting Review, XXXXVI (October, 1971), p. 701.

1 The production function is assumed to be a linear homogeneous one which, therefore, has the following properties

a Proportionality

b Additivity

c Divisibility

2 A linear cost function is assumed [35]

There also are two more assumptions which will "guarantee the existence and feasibility of a solution" but they will differ somewhat depending upon the orientation of the system being considered -- output or input

I Output oriented systems

3 Only one output may be produced by each process

4 "For each unit of output from any process the consumption induced in the same or prior processes must be strictly less than one unit "[36] 74

II Input-oriented system

3a Only one input may be consumed by each process

4a "For each unit of input to any process, the induced amount

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[35] *Ibid*, p 702. The three properties may be defined as follows

1 Proportionality outputs will increase by the same constant proportion as inputs

2 Additivity "input requirements for the sum of two alternative sets of outputs is identical to the sum of the inputs when computed for each output separately "

3 Divisibility fractional quantities are possible

[36] *Ibid*

of production ... that result in the same or subsequent processes must be strictly less than one unit.[37]

Appendix E presents the mathematical format for the traditional input-output model as developed by Leontief.

Input-Output Models and Standard Costs

Standard costs and the coefficients used in the input-output models tend to have several differences. The first such difference is in their construction:

> Standard costs are built upwards from the lowest basic operation while econometric parameters are broken downwards from aggregated material; standard cost data purport to illustrate the operation of the system, while econometric parameters are just weightings which happen to explain the right hand side of the equation in terms of the selected variable.[38]

The uses of standard costs also differ from those of the econometric (input output) coefficients; the former are used to forecast and control future performance; the latter only depict "an average of the actual expenditure in many production plants over an historical period of time."[39]

There are two possible ways in which input-output concepts could be used in conjunction with standard costing. The input-output tech-

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[37] Ibid., p. 712.

[38] Trevor E. Gambling and Ahmed Nour, "A Note on Input Output Analysis, Its Uses in Macro-Economics and Micro-Economics," The Accounting Review, XXXXV (January, 1970), p. 98.

[39] Ibid., p. 99.

nological matrix may be used to update standard costs, and it "is the only feasible way in very large systems of processes which are subject to continual change."[40] There will be many of the shortcomings of standard costing and mathematical programming which will not be obviated by such a process. However, one defect of traditional standard costing might be: the automated procedures which are used provide continuous updating of the data, a device which could be utilized to provide "automatic feedback of any cost and budget variances into the data bank itself"; thus, the standards could be continuously updated and used in the calculation and analysis of variances and a dynamic, rather than the traditionally static, situation would develop.[41]

A second way of employing the concepts of input-output analysis, one which is the primary concern of this chapter, is to use the input-oriented model as a means of distributing service department costs. This type of model has been discussed by several authors, including Williams and Griffin, Manes, Churchill, and Livingstone.[42] The general model, which will be discussed more fully in the following section, is set up for a situation in which service departments bill each other and the

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[40] Ibid. [41] Ibid., p. 102.

[42] Williams and Griffin, "Matrix Theory . . . ," pp. 134-144; Neil Churchill, "Linear Algebra and Cost Allocations," in Williams and Griffin, Management Information . . . , pp. 145-157; John Leslie Livingstone, "Matrix and Cost Allocations," The Accounting Review, XXXXIII (July, 1968), pp. 503-508; Rene Manes, "Comment on Matrix Theory and Cost Allocation," The Accounting Review, XXXX (July, 1965), pp. 640-643.

various producing departments for service rendered 'The cost of direct inputs to each process is given and the cost of the gross departmental outputs must be determined.'[43]

The coefficients of the technological matrix, productivity coefficients primarily, 'are functions of the levels of output' and 'reflect the proportionate amounts of dollar cost transferred from department j to department i.'[44] The determination of the coefficients may be achieved under either of two alternatives

1. ex post observations of the physical distribution of the services used to establish proportions based on actual utilization,
2. calculations of standard costs [45]

The ex post method suffers from a serious objection there is no base, or norm, against which the allocation percentages may be compared [46] This is, however, the technique used by Williams and Griffin, Manes, Churchill and Livingstone [47]

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[43] Butterworth and Sigloch p. 714.

[44] Ibid., pp. 714-715

[45] Ibid., p. 715, Livingstone, "Input-Output [illegible]" pp. 58-59 gives an example of how physical standards might be developed from the input-output model

[46] Butterworth and Sigloch, p. 715

[47] The respective articles were cited previously in footnote 42, page 158

Illustration of the Application of Input-Output Analysis[48]

The model to be discussed below is the same as that described on pages 151 through 154. Let A* represent the matrix of allocation percentages from department i to department j where a_{ij} is a typical element.

from / to	1	2	3	4	5	A	B	C
1	0	0	.05	10	20	0	0	0
2	0	0	10	05	20	0	0	0
3	10	10	0	05	20	0	0	0
4	05	0	10	0	20	0	0	0
5	10	10	05	0	0	0	0	0
A	.25	.80	20	0	10	0	0	0
B	25	0	30	40	05	0	0	0
C	25	0	20	40	05	0	0	0

= A*

This matrix is similar to that previously used, but it has been expanded to take into account the production departments. The letter B, as before, represents the vector of total costs to be distributed, but it also will be expanded to take into account the producing departments:

$$B^T = [8,000, \ 12,000, \ 6,000, \ 11,000, \ 13,000, \ 0, \ 0, \ 0]$$

Let A = I - A* where A represents the matrix of service department reciprocal cost allocation percentages subtracted from unity. The formula which will lead to the clearing of all service department costs into the producing departments will again be expressed as AX = B and $X = A^{-1}B$.

If it is desired, B may be broken down into its fixed and variable

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[48] Livingstone, "Input-output . . . ," pp. 50-51.

components in order to arrive at the allocations of each, i.e., let B' be the vector of fixed service department costs, then, $X' = A^{-1}B'$ would give the allocation of fixed costs and $X - X'$, the allocation of the variable components of the total cost.

Allocation of Joint Product Costs

Joint products are those products "that are necessarily produced together."[49] Joint costs, therefore are those costs which are necessary to take the joint products up to the split-off point and are not specifically related to any one of the co-products.[50] Some of the main reasons for allocating the joint costs between the several products are a need for cost for decision making and also a need to attach a cost to each product for inventory purposes.[51] If a standard cost system is being used, the distribution also is an aid in cost control.[52] This section will be concerned mainly with the latter two reasons - inventory costing and cost control.

There are two types of joint products which may be distinguished those which "are the output of fixed yield processes" and those which may give variable proportions.[53] In the former class, it is assumed

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[49]John S. Chiu and Don T. DeCoster, "Multiple Product Costing by Multiple Correlation Analysis," The Accounting Review, XXXXI (October, 1966), p. 673.

[50]Shillinglaw (3rd ed.), p. 233.

[51]Ibid.

[52]Ibid., p. 471.

[53]Ibid., p. 243.

that the percentage physical output of each co product is fixed by formula.[54] In the latter group, there are two situations which may arise

1. The type of joint materials used affects the percentage yield of each joint product
2. The processing methods employed can vary the relative yields of the joint products.[55]

The allocation of costs for fixed proportion joint products is felt to be impossible and "useless."[56] Because of this belief the statistical techniques to be discussed later in this section are directed mainly to the variable proportion, or "alternative" product case

Because of the usefulness of cost allocation for inventory costing, the traditional allocation techniques will be discussed, especially since they are applicable to both type of products For cost control purposes especially relating to the "alternative" products, it will be necessary to briefly discuss mix and yield variances and their analysis

Traditional Allocation Technique

There are at least two main methods which have been used in the allocation of joint costs to the separated products The first of these distributes the cost on the basis of same physical attribute of the product Such an allocation "assumes that the products should receive

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[54] Ibid

[55] Ibid

[56] Joel Dean Managerial Economics (Englewood Cliffs N J Prentice-Hall, Inc 1951), p 319 as quoted in Chiu and DeCoster p 675

costs relative to the benefits that the product received from the production process."[57] An example of how such an allocation might work would be to sum up all the units of each joint product and divide this grand total into the total joint cost This will yield an average unit cost applicable to all products [58]

There are several potential weaknesses in such a system The first shortcoming lies in the assumption that there is a direct proportionate relationship between the costs incurred and variations in the physical attribute being used for the allocation. Second, is the assumption that all the physical units are homogeneous this may not be the case.[59] These defects may be summarized into one main weakness the method ignores that cost-value relationship as long as the value of the total group exceeds the production cost, all joint costs are productive and thus, no product may be assigned a cost which exceeds its value [60]

The second broad allocation method is based on the ability of the products to absorb costs [61] There are two basic techniques of this method, depending upon the definition of market value being used

1 Relative sales value method under this procedure the allocation is based on the sales price of the products at the point of split-off The

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[57]Chiu and DeCoster p 674

[58]Shillinglaw (3rd ed.), p 236

[59]Chiu and DeCoster, p 674

[60]Shillinglaw (3rd ed), p 237

[61]Chiu and DeCoster, p 674

total market value of the batch is calculated and then the percentage of total market value for each co-product is determined. This percentage is then applied to the total cost to allocate it to the separate products.[62]

This technique, while generally eliminating the defects of the physical attribute method, also suffers from defects. It does not ensure that the allocated costs always will be less than market value or proportional to value.[63] This shortcoming arises because of the imperfection of using selling price at split-off as a measure of value, some products may have no market value at this point, but will have later on after further processing; others may have a value which is lower than their price due to high selling expenses.

II. Net realization basis. Net realization is "the selling price of the end product less any costs necessary to process it after the split-off point, sell it and distribute it."[64] The allocation procedure is the same as in the preceding method except that the joint costs are allocated based on the percentage of total net realization. The technique helps eliminate the problem of the relative selling price technique by the very definition of net realization.

Mix and Yield Variances

Traditionally mix and yield variances arise when one or more non-

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[62] Shillinglaw (3rd ed.), p. 238.

[63] *Ibid.*

[64] *Ibid.*

standard materials or labor groups are substituted for the standard materials or labor in a process and such substitutions bring about a change in the output of the process. These variances generally are contained within the traditional quantity variances, along with a third related "input quality variance" which arises when the substitution is of a higher or lower quality than the standard input. The total quantity variance may be broken down as follows:

(1) Actual cost (at standard prices)
(2) Standard Cost for standard mix, standard quality
    Yield variance (1) - (2)
(3) Standard Cost for standard mix, actual input quality
    Input quality variance (2) - (3)
(4) Costs earned (actual mix)
    Operating mix variance (3) - (4)
Total Variance (1) - (4) [65]

The analysis of such variances is relevant for both the fixed and variable proportion joint products. In the case of fixed proportion joint products, the change from a standard input to a nonstandard one may affect the total output as well as each of the individual product outputs based on their normal proportions. In the variable proportion case the inputs may be changed from the standard mix intentionally in order to obtain a particular effect on the yield. The mix and yield variances are still appropriate since it would be necessary to measure the difference between the standard and actual costs. In such a situation, it may still be desirable to compute the variance between the old standard mix

[65] _Ibid._, p. 474.

and the m[illegible] but a more interesting set of variances would arise if the standard were initially changed to take into account the new mix and then this updated standard were used as the point of reference against which to compare the actual results

Recent developments

Not much has been written in regard to the mix and yield variances as far as their analysis in terms of a statistical or management science procedure is concerned Hasseldine has expressed the variances in terms of mathematical formulae and analyzed them graphically, but he has not carried the analysis further.[66] Another discussion of these variances in terms of mathematical formulae has been presented by Gordon [67]

Wolk and Hillman, in a more recent article, employ a different approach [68] First, they use a linear programming model to determine the optimal short run combination of raw materials, the only input anal

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[66] C. R Hasseldine "Mix and Yield Variances " The Accounting Review XXXXII (July 1967), pp 497-515

[67] Myron J Gordon "The Use of Administered Price Systems to Control Large Organizations " in Management Controls - New Directions in Basic Research, Eds Charles P, Bonini Robert K Jaedicke, and Harvey Wagner (New York McGraw Hill Book Company, 1964) pp 16-17

[68] Harry L Wolk and A Douglas Hillman, "Materials Mix and Yield Variances: A Suggested Improvement " The Accounting Review, XXXXVII (July, 1972) pp 549-555

yield in the example. From the [illegible] of the model, traditional mix and yield variances are calculated when it is necessary to use a different input mix than that given by the optimal solution. To make these variances more meaningful, especially in those cases where the standard mix has been purposely abandoned, they should be calculated using the new short run optimal mix for actual production.

Multiple Correlation Analysis

This section will first give a general discussion of the background on the need for marginal costing. Then there will be a brief discussion of how incremental costs might be determined for "alternative" products under conventional methods, and finally, the application of multiple correlation analysis to this problem.

Background

To use an approach such as multiple correlation for joint cost allocation requires a shift in thinking on the part of the firm. Rather than view co-product costs in relation to their significance to the firm, they should be considered on the basis of how they were generated.[69] It is necessary to determine if the products are truly joint, i.e., increased production of one leads to increases in all others, or if they are alternatives, i.e., production of one reduces the output of the others.[70] If the latter is the true situation, a more useful cost analysis through

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[69]Chiu and DeCoster, pp. 674-675.

[70]Ibid.

marginal costs is possible. "The cost of an alternative product can always be computed in terms of the foregone profits from the other product."[71]

Incremental costing

It was mentioned on page 162 that there are two situations for variable proportions which may arise when analyzing joint costs. Because of the ability of relative yields to vary, it is possible to measure the incremental costs to which such variations give rise.[72] The determination of such marginal costs is easiest to determine when the yield is affected by the type of materials used; all that is necessary is to look at the changed outputs and costs.[73] The procedure is similar, although more complex, if the yield is altered by changing the method of processing. In this situation the incremental costs equal "the sum of incremental processing costs plus the sales value of the other joint products lost by the shift in the product mix."[74] This type of an approach has two prime defects: 1) Incremental cost is variable, depending on the relative yields; the approach would require a table of incremental costs for various product mixes. 2) The opportunity cost,

[71]Dean, p. 319, as quoted in Chiu and DeCoster, p. 675.

[72]Shillinglaw (3rd ed.), p. 243.

[73]*Ibid.*

[74]*Ibid.*, p. 244.

the foregone profit, may not equal the product price [75]

<u>Application of multiple correlation analysis</u>

Multiple correlation aids the accountant in determining the marginal costs which generally are not provided by the traditional methods of cost allocation [76] As a technique to be used for this purpose multiple correlation should be viewed in terms of both its advantages and its limitations which generally are due to the underlying assumptions of the model

I Advantages Multiple correlation enables the analyst to simultaneously estimate the marginal cost of all multiple products because it recognizes the cost structure of such products [77] It is primarily a *ceteris paribus* approach in that the effect of only one change in output is viewed in determining the marginal cost [78]

II Limitations A number of constraints affect the applicability of multiple correlation analysis to the multiple product costing problem

1 Product limitations only joint products which fall into the variable proportion category may be costed using these techniques

2 Equation limitations the ability to find the right model linear or nonlinear, will affect the reliability of the estimates.

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[75] *Ibid*

[76] Chiu and DeCoster, p 675.

[77] *Ibid* , p 677.

[78] *Ibid*

3. Period limitations:

   a. The data are viewed at specific points of time, not 'over a continuum' as traditionally assumed which means that any statistical measure which is derived will be an average

   b. The relationship between variables can be extrapolated only over the range of observations

   c. The number of observations required is very large

4. Causation limitations the technique cannot identify cause and effect relationships [79]

__Example__

The following example, using multiple linear regression and multiple correlation, was presented by Chiu and DeCoster.[80] In this example they dealt with a firm which produced three alternative products at a total cost, Y. Observations for ten periods were used in establishing the model which was formulated as

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3$$

The coefficient b_0 of the model represents the standby cost which is incurred at zero output. The linear marginal costs for the three products (X_1, X_2, X_3) are determined as the other coefficients in the model b_1, b_2, b_3. Standard errors of the estimate are also determined for each of these cost estimates, S_{b_1}. The multiple correlation coefficient,

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[79] _Ibid._, pp. 678-679

[80] _Ibid._, pp. 675-678.

R, and the coefficient of multiple determination $D (= R^2)$, are also computed.

The size of R helps in establishing the validity of the model, e.g., the closer R is to 1.000, the more valid is the linear model being used. The size of D shows the percent of variation in total cost which is explained by the three products acting together. The significance of each product is determined from the t-ratios $t_i = \frac{b_i - \beta_i}{S_{b_i}}$ where β_i represents the true linear marginal cost. The confidence limits for β_i may be determined from $\beta_i = b_i + t_{n-k-1} S_{b_i}$ where (n-k-1) is the degrees of freedom to be used in finding the range of t which is determined from a table. This range establishes the limits between which t_i should vary if the assumption on β_i being used is true. If t_i is outside this range, the assumption about β_i is not true and the total cost, Y, will be dependent upon the output level of X_i. The linear marginal cost, β_i, of this product will fall within the confidence limits established for β_i but the actual value is indeterminate.

A similar example is described by McClenon in which he looks at a situation where total costs are known over a period of time as well as the physical quantities of the different types of products which are processed.[81] Multiple regression analysis is used to find the individual unit costs for each product. McClenon's approach differs somewhat

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[81]Paul R. McClenon, "Cost Finding Through Multiple Correlation Analysis," The Accounting Review, XXXVIII (July, 1963), pp. 540-547.

from the first example in that he does not carry the analysis to the point of determining S_{b_j}, R, D and t, but works only with "least squares estimates of unit costs by types."[82] He recognizes the need to calculate these additional statistics but feels they should be set aside temporarily until accountants become more familiar with the use of statistical tools.[83]

Impact on Standard Costs

Standard costs for joint products traditionally are computed using a "relative market value" approach which may be set up in the following format.[84]

Grade of output	(1) Standard amount of output per X amount of input	(2) Estimated market price per unit of output	(3) Market value per X amount of input	(4) Percent of total market value	(5) Total cost allocated	(6) Standard cost per unit of output

This approach may be appropriate for the fixed proportion joint cost situation but not for the variable proportion case. The same problem would arise as in the incremental costing situation: a different set of standards would have to be set up for all possible mixes.

The multiple correlation approach will help in the alternative product case in that it is useful in obtaining unit cost estimates, standard and actual, for the individual outputs from the joint processing. The

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[82] Ibid., p. 543. [83] Ibid.

[84] Shillinglaw (3rd ed.), p. 471.

resulting unit costs are averages over the time period of the observations which helps to eliminate the problems caused by the varying relative yields which lead to the changing incremental costs. Such an average may be used as the basis for constructing an expected actual standard cost for each product which may then be used as a benchmark for variance analysis.

The changes which have been suggested for the mix and yield variances operate mainly to clarify the concepts involved by means of their expression purely in mathematical form and to try to alleviate the difficulties caused by interpreting them, especially when the variance arises due to a planned change in the input mix.

Summary

Two separate types of cost allocation problems have been discussed: the allocation of service department costs to producing departments when reciprocal relationships between the service departments exist; and the allocation of joint costs at the split off point to the various "alternative" products. In both instances the traditional procedures for distributing the costs were described first and then the proposed methods. In the case of service departments, the proposed methods were in the area of management science -- matrix algebra and input-output analysis, whereas statistical methods, multiple correlation analysis in particular, were suggested for the joint product case.

The impact of the statistical and management science techniques on

standard costing of these allocation questions is more indirect than the previous areas discussed, e.g., learning curves, control charts and linear programming models. The matrix algebra techniques are only a computational device to facilitate the distribution of the service department costs. The multiple regression results provide a way of allocating a total cost to various outputs and generally the figures involved in the analysis will be the actual costs to be used in variance analysis or will provide an historical basis upon which the expected actual standard will rest.

VII SUMMARY AND FUTURE PROSPECTS

A number of statistical and management science techniques which have the potential of seriously affecting standard costing were discussed in the preceding chapters along with their impact, realized or potential. The techniques which were considered have been those generally involved in the construction of standards, the analysis of variances and the allocation of costs either among departments or between products.

The basic view of a standard as a benchmark has been altered by many of the techniques discussed. Traditionally the standard, price and/or quantity, was, and often still is, viewed as a single, static point estimate. Control charts have abandoned this concept in favor of a range of costs bounded by the control limits. This type of thinking has influenced the interpretation of variances in standard cost control situations. For product costing purposes the standard may be viewed as similar to the mean of the control chart. Modern decision theory techniques have suggested that both of these views be replaced by an expected value type of standard. Controlled cost replaces the point estimate and range of costs with a probability distribution. Learning curves although based upon the future attainment of a predetermined standard, provide a means of automatically updating the expected

standard as learning occurs, it makes the process of setting the standard dynamic.

This group of statistical techniques have had one common impact upon the concepts involved in developing standards, namely, that one need not be bound by the traditional view of the single benchmark, but may, if circumstances warrant, use some alternative technique to arrive at an appropriate construct.

Two specialized procedures were discussed which also were involved in the construction of standards, although less directly than the preceding techniques. The first of these, the division of mixed overhead costs into their fixed and variable cost components, utilized regression analysis, generally in conjunction with correlation analysis. The result was a mathematically determined separation which removed much of the subjectivity of the traditional techniques. The addition of correlation analysis was felt to be useful in the choice of the appropriate independent variable to be utilized in the construction of fixed and variable overhead rates per unit. The second technique was in the area of the development of data inputs for linear programming models. While traditional standards were felt to be adequate as first approximations, it was suggested that they be modified so as to remove any tightness built in for motivational purposes or, in the case of standard costs, to ensure that all of the relevant costs for a particular item are included. In addition, sensitivity analysis may be used to test the range in which the inputs may vary before a given solution is no longer

optimal

The latter of these techniques ties in with the general impact of the statistical methods mentioned above. Traditional standards need to be modified to enhance their usefulness and a range of permissible fluctuation established. The major impact of regression analysis lies in its role as an improved computational technique to be used in the construction of traditional overhead standards. The resulting separation may establish the fixed cost and rate of variability more precisely than was the case with traditional accounting methods of separation.

Variance analysis has also been affected by many of the techniques discussed in the preceding chapters. The guiding principle in this area has been, and still is, management by exception. Various statistical techniques have attempted to improve the differentiation among the variances to determine which ones are the most beneficial for management to investigate. Control charts and modern decision theory both differentiate between those deviations due to controllable factors which are to be investigated and those occurring from random noncontrollable events. This helps to limit the number of variances which are reported to management for corrective action. In addition, modern decision theory techniques consider the costs involved in investigating, or failing to investigate, a particular variance. While this latter step also may limit the number of deviations felt to be worth investigating, it may also highlight some variances which the other techniques pass over because they fall within the control limits. An additional im-

provement related to the control function which is brought about by the utilization of control charts or modern decision theory is the increased timeliness of reporting of variances to management. This has occurred because of the more frequent data collection necessitated by statistical procedures. Control charts, at least, may be adapted also to take into consideration learning and thus reduce the effects of learning in the analysis of variances.

While these statistical techniques do not act explicitly to improve the variances which are calculated, they do improve the ability to make the decision as to whether or not an investigation is warranted, especially in those situations which utilize modern decision theory approaches. They also improve the detection of significant variances because of the more frequent data collection which reduces the possibility of their being averaged out over time. Control charts also offer several warning signals that a system may be operating out of control even though all of the variances are occurring within the limits. Because of its relative simplicity, the control chart appears to have gained more acceptance than modern decision theory approaches. Controlled cost, which looks at the investigative decision in terms of a decision as to whether the actual and controlled cost are from the same universe, is a technique which may have a potential impact to be determined only after additional research.

The linear programming approach to variance analysis looked at the problem from a somewhat different point of view. Allowable vari-

ances in the data inputs are determined after the optimum solution is derived and the effect of such variances upon the "figure of merit" is analyzed by means of the shadow prices, opportunity costs developed as a part of the solution. It is possible, with linear programming, to take into account many of the individual factors which normally are included in the aggregate figures used in the traditional analysis, e.g., for a material price variance: substitute products, price fluctuations, inflation, etc. The complete impact of the use of linear programming and resultant opportunity costs upon the analyses of variances does not appear to have been fully explored at this time.

The final general area of standard costing which was discussed related to the impact of statistical and management science techniques upon cost allocation, a term covering two separate topics: service department cost allocation and allocation of joint costs among co-products. Matrix algebra and a related technique, input-output analysis, were suggested for use in the allocation of service department costs to production departments where reciprocal relationships exist. The only impact which may be attributed to these techniques is that they simplify necessary computations once the initial inverse matrix of the allocation percentages is obtained.

Regression analysis has been suggested as an improved technique for allocating costs among variable proportion co-products. It helps in arriving at average unit costs for individual outputs over a given period of time. These averages, then, may be used to develop the standard

costs to be employed for a variety of purposes. The main thrust of this technique, therefore, is in the direction of eliminating the need to establish a set of standards for each projected product mix.

Statistical and management science techniques which have been discussed in the preceding chapters have had a varied immediate impact upon both the construction and utilization of standard manufacturing costs. For many of these techniques the impact is more potential than realized because of a lack of general acceptance, e.g., the use of linear programming results for variance analysis. Two possible reasons for the slow acceptance of several of the proposed techniques may be the view that they require specialized knowledge and/or computers. As accountants continue to realize the need to expand their understanding of various statistical and management science techniques, the first of the above reasons given for an unwillingness to use more complex techniques should become less valid. The need for computers and related software exists to implement the techniques of regression analysis, matrix algebra, and linear programming in particular. The widespread availability of computers should make the need for their usage an invalid reason for failure to employ these techniques.

Future Prospects

If it may be assumed that the future may be viewed as an extension of the past, then it becomes relatively easy to forecast, in the light of this study, tendencies in the evolution of standard costing in the coming

decade As has been indicated, the history of standard costing is replete with much borrowing of techniques of analysis from other disciplines -- scientific management, statistics and management science, in particular There is no reason why this process should not continue

In the past many studies have appeared in the literature advocating the application of various statistical and management science techniques to standard costing situations Articles of this nature will, undoubtedly, continue to appear. Some of the techniques mentioned have been included in cost accounting texts This trend should continue and expand, as time goes by

Research development, it appears, may proceed along two lines. Some research will be aimed at elaborating and expanding upon the techniques discussed in the preceding chapters and where feasible, attempt to make them operational in accounting practice In addition, other techniques of statistics and management science which are felt to be closely related conceptually to standard costing and its uses may receive attention Examples of such techniques include PERT, curvilinear statistical models and various nonlinear mathematical programming techniques (e g , integer, piece-wise linear, quadratic) which would permit more realistic approximations of the cost and production function existing within a firm

Several uses of standard costing were mentioned in the first chapter and a number of ways of constructing the standards have been reviewed varying from a point estimate to a range of costs and to an ex

pected value concept. Some of these standards may be more applicable to one use than to the others, e.g., one would tend to use a point estimate for inventory costing or pricing but a range of costs or expected value concept of standards might be more appropriate for variance analysis and, further, a modified point estimate, adjusted for various factors, is more suitable for linear programming. As more statistical and management science techniques are adopted, the possibility of constructing a series of standards for each cost item -- price and quantity -- to serve a variety of possible uses should be considered. Such a series might be developed in the form of a vector for easy computer storage.

The area of possibly the greatest potential for future research lies in the analysis of behavioral implications on performance or motivation of many of the techniques which are currently in use or have been advocated for adoption. This topic has only been mentioned in passing in this study. The results of such research may affect the adoption of many of these techniques into general practice. As the techniques utilized in standard costing become more complex mathematically, they may no longer be capable of permitting the desired feature of participation in standard construction which is felt to be essential to the acceptance of a procedure and its results. Some research has already been done in this area in connection with gaining the acceptance of control charts for variance analysis, but more is needed.

APPENDICES

Appendix A   Example of a Cost Estimating Procedure[1]

Explanations

1) Complexity analysis involves the estimation of the labor hours of the proposed product from the actual hours involved in a similar, previously produced item.[2]

2) The standard cost data are utilized in order to determine at which future unit the standard will be attained.[3]

3) The base unit man hours are determined from a combination of formulas, complexity analysis and standard cost data. Only one may be relevant to a particular situation, but the other methods

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[1] Cochran, *Planning Production Costs*, p. 204.

[2] *Ibid.*, p. 252

[3] *Ibid.*, p. 257

may b[illegible] as a cross-check.[4]

4) The slope may be affected by two factors: learning and progress. The learning aspect is affected by the amount of mechanical control which exists over the operation and this would be the point at which to start determining the slope.[5]

Progress refers to a reduction in labor hours for one or more of the following reasons:

Increased Lot Size
Improved Methods (major)
Substituted Material
Mechanized Existing Processes
Loosened Quality Standards
Developed New Processes
Simplified Design[6]

The effects of these during-production factors generally are determined from historical time data and the careful study of why the reductions occurred in the past.[7]

5) The C(n) tables are derived from the basic formula: $C(2n) = C(n)2^{-b}$ where C(n) is the time related to the production of the nth unit, and b is a constant relating to the slope. The tables which are developed show the value of the nth unit's time as related to some other unit for different values of b.[8]

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[4]*Ibid.*, p. 251.
[5]*Ibid.*, p. 212.
[6]*Ibid.*, p. 222.
[7]*Ibid.*, pp. 223-224.
[8]Cochran, "New Concepts [illegible]," p. 318.

Appendix B Comparative Example of Variance Analyses

The following discussion is a comparison of the variance analysis procedures suggested by Gillespie in 1935 and those used at the present time, as described by Horngren.[1]

The variances which appear to be calculated in the same fashion by both authors are those relating to material and labor:

1. Material price variance (actual price - standard price) x actual quantity (in terms of purchases or usage)
2. Material quantity variance (actual material used - standard quantity allowed) x standard price
3. Labor price (rate) variance (actual rate - standard rate) x actual hours
4. Labor quantity (efficiency) variance (actual hours used - standard hours allowed) x standard rate

The major difference in the techniques arises when comparing the analysis of overhead variances. This may be attributed mainly to the fact the Gillespie did not separate the overhead costs into their fixed and variable components and analyze the variances of each type of cost

[1]Gillespie (1935) pp. 27-29; Horngren, p. 246.

separately. If one were to break the total costs into their components when utilizing Gillespie's method, however, some similarities in the results would become apparent. The following numerical example applies to the analysis of overhead variances as suggested by Gillespie and Horngren.

	Budget		Actual
Direct labor hours	1,000		1,100
Units	500		525
	per hour	total	
Variable cost	$ .90	$ 900	$1,050
Fixed cost	.60	600	650
Total	$1.50	$1,500	$1,700

Gillespie's Technique

	Actual (1)	Budget (2)	Standard cost x actual hours (3)	Standard cost x standard hours allowed (4)
Variable cost	1,050	900	990	945
Fixed cost	650	600	660	630
	1,700	1,500	1,650	1,575

(1) - (2) $200 budget (price) variance unfavorable

(2) - (3) $150 idle time variance unfavorable

(3) (4) $ 75 quantity variance unfavorable

Horngren's Procedure

	Input actual cost (1)	Input budget actual hours (2)	Output budget, standard hours allowed (3)	Overhead applied standard hours allowed (4)
Variable cost	1,050	990	945	945
Fixed cost	650	600	600	630
	1,700	1,590	1,545	1,575

(1) - (2)	$110	spending variance	unfaverable
(2) (3)	$ 45	efficiency variance	unfaverable
(3) - (4)	$ 30	volume variance	unfaverable

That the differences in the figures are due to the failure to separate the costs into their two components becomes readily apparent. For example Gillespie's budget variance is composed of more than Horngren's spending variance because of the use of the fixed budget

Appendix C [illegible][1]

The following is a summary of an example presented by Samuels. The firm being studied, a decentralized firm, produces three products, X, Y and Z, which require the use of three scarce resources; floor space, supervision, and machines. The contribution margins (unit selling price less unit marginal cost) for the products are $2, $3 and $4, respectively. (For typing ease the symbol for pounds, as used by Samuels, has been replaced by the dollar sign.) The problem is set up to determine two things:

1. the amounts of the products to be produced which will yield the maximum profit.
2. the optimal allocation of the scarce resources to the departments (one for each product) which will make their operation harmonious with the goals of the firm as a whole.

<u>Initial Problem</u>

Maximize $\quad 2X + 3Y + 4Z$

Subject to

$$\begin{aligned} 5X + Y + Z &\leq 8{,}000 \quad \text{floor space} \\ X + 5Y + Z &\leq 8{,}000 \quad \text{supervision} \\ X + Y + 5Z &\leq 8{,}000 \quad \text{machines} \end{aligned}$$

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[1] Samuels, pp. 184-189

Initial Tableau

prices	2	3	4	0	0	0	b	C_B
products	X	Y	Z	S_{L1}	S_{L2}	S_{L3}		
	5	1	1	1	0	0	8,000	0
	1	5	1	0	1	0	8,000	0
	1	1	5	0	0	1	8,000	0
$Z_j - C_j$	-2	-3	-4	0	0	0	0	

The S_{Li} (i = 1, 2, 3) are the respective slack variables necessary to make the constraining inequalities into equations. The $Z_j - C_j$, especially in the optimal solution, represent the "per unit opportunity cost of bringing a variable into the solution."[2]

Optimal Tableau

prices	2	3	4	0	0	0	b	C_B
products	X	Y	Z	S_{L1}	S_{L2}	S_{L3}		
	1	0	0	3/14	-1/28	-1/28	1,142	2
	0	1	0	1/28	3/14	-1/28	1,143	3
	0	0	1	-1/28	-1/28	3/14	1,143	4
$Z_j - C_j$	0	0	0	5/28	12/28	19/28	10,285	

This result provides information which may be useful in analyzing two separate situations:

A. Production of output not equal to the budget[3]

Under traditional standard costing, this situation would lead to unabsorbed overhead and an unfavorable "volume variance." Under

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[2] Ibid., p. 185.

[3] Ibid., pp. 185-186.

Samuels' method the "real loss" may be measured. Assume only 800 units of X were produced. Its producing department would incur a loss of $684 (342 units x $2 per unit, where the $2 represents the opportunity cost).

If, instead, 1,183 units of X were produced and 1,143 units of Y, the amount of Z which could be produced would be affected by the over-production of X as follows:

product	output	units of floor space	total
X	1,183 x	5	5,915
Y	1,143 x	1	1,143
Z	942 x	1	942
			8,000

The department producing X would be charged with the difference between the optimal contribution less the contribution actually achieved, or

product	optimal contribution	actual contribution	difference
X	1,142 x 2 = 2,284	1,183 x 2 = 2,366	82
Y	1,143 x 3 = 3,429	1,143 x 3 = 3,429	0
Z	1,143 x 4 = 4,572	942 x 4 = 3,768	-804
	10,285	9,563	-722

B. Transfer pricing

In this case the shadow prices, $Z_j - C_j$, are used as the basis of the standard cost system.[4] These prices may be used "to charge each department for the use of the scarce resources."[5] The departments will break even only when they use the budgeted amounts, thus

[4] Ibid., p. 186.

[5] Ibid.

[illegible]	floor space	supervisors	machines	contribution margin
X	5(5/28)	1(12/28)	1(19/28)	2
Y	1(5/28)	5(12/28)	1(19/28)	3
Z	1(5/28)	1(12/28)	5(19/28)	4

The following table summarizes the use of the opportunity costs and shadow prices. As an additional assumption, the units of supervisor time in department X will be 986 rather than the higher (standard) amount determined for its output

	units	price	cost of using resources
Product X			
Floor space	5, 915	5/28	1 056 2
Supervisors	986	12/28	422 5
Machines	1, 183	19/28	802 8
			2, 281 5
Opportunity cost transferred			722 0
			3, 003 5
Contribution margin earned (1, 183x$2)			2, 366 0
Less			637 5
Product Y			
Floor space	1, 143	5/28	204 1
Supervisors	5, 715	12/28	2, 449 3
Machines	1, 143	19/28	775 6
			3, 429 0
Contribution margin earned (1, 143x$3)			3, 429 0
Balance			0
Product Z			
Floor space	942	5/28	168 2
Supervisors	942	12/28	403 7
Machines	4, 710	19/28	3, 196 1
			3, 768 0
Contribution margin earned (942x$4)			3, 768 0
Balance			0

The department producing X has saved $84 5 in supervision budgeted cost of outputs achieved (9, 563) less opportunity costs from the product accounts (9 478 5) as is seen from the following accounts.

CONTROL ACCOUNTS[6]

Contributions

Optimal Contribution, per budget	10,285	Contributions Earned from product accounts	
		Dept X	2,366
		Dept Y	3,429
		Dept Z	3,768
			9,563
		Balance being lost opportunity charged to Department X	722
	10,285		10,285

Costs

Opportunity Costs from product accounts		Budgeted Costs on outputs achieved[a]	9,563
Dept. X	2,281.5		
Dept Y	3,429.0		
Dept. Z	3,768.0		
	9,478.5		
Balance, being saved of Dept X[b]	84.5		
	9,563.0		9,563

Reconciliation

Missed Opportunity charged to Dept X	722	Saving of Dept X in use of Supervisors	84.5
		Balance, being loss on cost accounts[c]	637.5
	722		722.0

[6] Ibid., p. 169

NOTES ON ACCOUNTS[7]

a The budgeted opportunity cost of a department on the output achieved is equal to the contribution earned by that department. This is the result of charges based on shadow prices, that is, departments are budgeted to break-even.

b The saving of dept. X on supervisors is calculated as follows:

Inputs per budget on output of 1,183 units	1,183
Actual inputs	986
	197

That is, 197 units of supervisors' time at a shadow price of 12/28 84.5

c The balance in the reconciliation account is the loss of Dept. X; the other two departments break-even.

[7]*Ibid.*, p. 189.

Appendix D Some Examples of Ex Post Analysis[1]

Mathematical Notation

Because three sets of results are used in this model, the superscripts a, o, and p are used to denote the ex ante, observed, and ex post results respectively. Total net income for the period, regardless of the result being used, is determined by: $NI = CX - F$, where X represents the output vector and F the total fixed costs. The formula for analyzing variances will be expressed as

$$NI^a - NI^o = (NI^a - NI^p) + (NI^p - NI^o)$$

where $(NI^a - NI^p)$ represents the forecasting error

$(NI^p - NI^o)$ provides the opportunity cost

Two Examples

Initial problem

$$\begin{array}{llllll} \text{Maximize} & 1.2X_1 + & 1.1X_2 + & 1.0X_3 & & \\ \text{Subject to} & X_1 + & X_2 + & X_3 & \leq 300 & \\ & X_1 + & X_2 + & X_3 & \leq 200 & \\ & X_1 + & X_2 + & X_3 & \leq 200 & \\ & & & X_i & \geq 0 & i = 1, 2, 3 \end{array}$$

The coefficients of the objective function represent the contribution

[1]Demski, Variance Analysis, Chapter IV

margins of the products,

Optimum Tableau

prices	1.2	1.1	1.0	0	0	0	b	C_B
products	X_1	X_2	X_3	X_4	X_5	X_6		
	0	0	1	1	1	0	100	1.0
	1	1	0	0	1	0	200	1.2
	0	1	1	-1	1	1	100	0
$Z_j - C_j$	0	1	0	1.0	2	0		

$(X^a)^T = (200,\ 0\ \ 100,\ \ 0,\ \ 0,\ \ 100\ )$ $\quad C^a X^a = 340$ [2]

Example 1 unfavorable material price perturbation[3]

Let the unfavorable material price perturbation be 30 units of product X_1. The observed contribution margin will be 0.9 as opposed to the 1.2 ex ante amount. The only change in the parameters of the problem will be in the C vector $C^o = (0.9,\ 1.1,\ 1.0,\ 0\ \ 0,\ 0)$.

1. If the perturbation were avoidable: $C^p = C^a$, $NI^p = NI^a$ and $NI^o = C^o X^a = 280$. The variance would be determined as

$$NI^a - NI^o = (NI^a - NI^p) + (NI^p - NI^o)$$
$$= \quad 0 \quad + \quad 60$$

which is the opportunity of the perturbation.

2. If the perturbation were unavoidable, $C^p = C^o$ and, after inserting C^p into the final tableau and resolving the problem, $(X^p)^T = (100,\ 100,\ 100,\ 0,\ 0,\ 0)$ would be the new solution

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[2] Ibid., pp. 48-49.

[3] Ibid., pp. 49-50.

vector with $C^P X^P = 300$. The variance in this case is

$$NI^a - NI^o = (NI^a - NI^p) + (NI^p - NI^o)$$
$$= (340 - 300) + (300 - 280)$$
$$= \quad 40 \quad + \quad 20$$

where the 40 units represent the forecasting error and the 20 units the opportunity cost. Traditional variance analysis would have arrived at a deviation of 60 units (340 - 280).

<u>Example 2 the handling of a joint product term</u>[4]

The same <u>ex ante</u> program will be used as in the preceding example but the changes in the observed results will be more extensive $C^o = C^a$, $(X^o)^T = (100, 0, 100, 100, 0, 0)$, $C^o X^o = 220$, $b^o = b^a$, the vector of constants and A^a, the matrix of technical coefficients becomes

$$A^a = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1/2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

The <u>ex post</u> results are $C^P = C^a = C^o$, $A^P = A^a$, $b^P = b^a = b^o$, $(X^P)^T = (50, 200, 0, 50, 0, 0)$ and $C^P X^P = 280$. The variance in the net income can be determined as

$$NI - NI = (NI - NI) + (NI - NI)$$
$$= (340 - 280) + (280 - 220)$$
$$= \quad 60 \quad + \quad 60$$

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[4]<u>Ibid</u>., pp. 52-54.

If the following assumptions are made, the above results may be broken down further:

1. each change in the a_{ij} was due to a labor efficiency perturbation,
2. the wage rate for process two equalled 2 units and for process three 3 units,
3. there were favorable direct material perturbations for products X_1, X_2 and X_3 of 1, -1.5 and 2 units respectively;
4. the wage rate perturbation in process two was 0.5 units, favorable
5. the perturbations were unavoidable

If traditional accounting variances were calculated, the following would occur

1) Price and efficiency variance $(C^a - C^o)X^o$

X_1:	material price variance	1(100)	100 F
	labor use variance	1(100)(2)	200 U
	wage rate variance	.5(200)	100 F
			0

X_3:	material price variance	2(100)	200 F
	labor use variance	1(100)(3)	300 U
	wage rate variance	.5(200)	100 F
			0

2) Mix and volume variances $C^a(X^a - X^o)$

$100(C_1^a)$ 100(1.2)   120 U

In contrast, the <u>ex post</u> analysis would yield the following:

1) Forecasting variances $(NI^a - NI^p)$

a) Basis variances	$C^p(X^a - X^p)$	60	U
b) Price and efficiency variances	$(C^a - C^p)X^a$		
X_1: material price variance	1(200)	200	F
labor use variance	1(200)(2)	400	U
wage rate variance	.5(400)	200	F
		0	
X_2: material price variance	1.5(0)	0	
labor use variance	.5(2)(0)	0	
wage rate variance	.5(1)(0)	0	
		0	
X_3: material price variance	2(100)	200	F
labor use variance	1(100)(3)	300	U
wage rate variance	.5(200)	100	F
		0	

2) Nonoptimal utilization variance $(NI^p - NI^o)$

a) Basis variance	$C^p(X^p - X^o)$	60	U
b) Price and efficiency variances	$(C^p - C^o)X^o$	0	

Appendix E Mathematical Form of the General Input-Output Model*

Let T represent the transactions matrix (n x n) in which there is one row and one column for each activity. The typical element, a_{ij}, will represent the amount or value, of the output of the ith activity which has been used as an input to the jth activity, the rows represent the uses of outputs and the columns, the sources of inputs

There also are two (n x 1) columns: one which shows the final demand for each activity, b_i, and the other, the total output, x_i, and a (1 x n) row which displays the costs of the primary input to the activities, e_j, and the total cost of this input, W

The transactions matrix

$$\left[\begin{array}{c:c:c} a_{ij} & b_i & x_i = b_i + \sum a_{ij} \\ \hdashline e_j & 0 & W = \sum e_j \end{array}\right]$$

From this matrix it is necessary to compute the technological coefficients $a_{ij} = a_{ij} / E_j$ where $E_j = e_j + \sum_i a_{ij}$ which are then used to derive an input coefficient matrix: $A^* = \{a'_{ij}\}$. This new matrix also is (n x n). The A* matrix will be used to develop the technology matrix

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*Livingstone, "Input-Output . . . ," pp 51-53

$A = I - A'$, where I is the identity matrix

$$\begin{bmatrix} 1 & -a'_{12} & \cdots & -a'_{1n} \\ -a'_{21} & 1 & \cdots & -a'_{2n} \\ \vdots & & & \vdots \\ -a'_{n1} & -a'_{n2} & \cdots & 1 \end{bmatrix} = A$$

The solution to the system is determined from $Ax = b$ or $x = A^{-1}b$, which says that all the outputs have been distributed over all uses whether final or intermediate

The e_j's must be determined indirectly after the final demand has been calculated, thus for column c, $e_c = x_i(1 - \sum_i a'_{ic})$ where x_i is derived from two conditions $x = A^{-1}b$ and the fact that $x_i = x_j$ for all $i = j$ (total inputs equal total output for each activity) The term $\sum_i a'_{ic}$ is given in the calculation of the technological coefficients

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy

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